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► **To cite this version:**

Guy Royer. Fusion, fission, alpha emission and superheavy element formation and decay within a generalized liquid drop model. French-Japanese Symposium on Nuclear Structure Problems, FJSNSP-LIA, Sep 2013, Paris, France. in2p3-01151846

**HAL Id: in2p3-01151846**

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Submitted on 13 May 2015

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# Fusion, fission, alpha emission and superheavy element formation and decay within a generalized liquid drop model

**Guy Royer**

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- **Generalized Liquid Drop Model**
- **Fusion**
- **Fission**
- **Alpha decay**
- **Superheavy nuclei**

# Liquid Drop Model energy

$$E_{\text{GLDM}} = (E_{\text{vol}} + E_{\text{surface}} + E_{\text{Coulomb}} + E_{\text{proximity}}) + (E_{\text{shell}} + E_{\text{pairing}})$$

$$E_{\text{vol}} = -a_v (1 - k_v I^2) A$$

$$E_{\text{surf}} = a_s (1 - k_s I^2) A^{2/3} \times \frac{\text{Surf}}{4\pi R_0^2}$$

$$E_{\text{Coul}} = \frac{9e^2 Z^2}{16\pi^2 R_0^6} \iiint \frac{d\tau d\tau'}{|\vec{r} - \vec{r}'|}$$

$$E_{\text{shell}} = E_{\text{shell}}^{\text{sphere}} (1 - 3.1\theta^2) e^{-\theta^2}$$

$$E_{\text{pairing}} = E_{\text{TF model}}$$

$$I = (N - Z) / A$$

$$a_v = 15.494 \text{ MeV}$$

$$a_s = 17.9439 \text{ MeV}$$

$$k_v = 1.8$$

$$k_s = 2.6$$

$$R_0 = 1.28A^{1/3} - 0.76 + 0.8A^{-1/3}$$

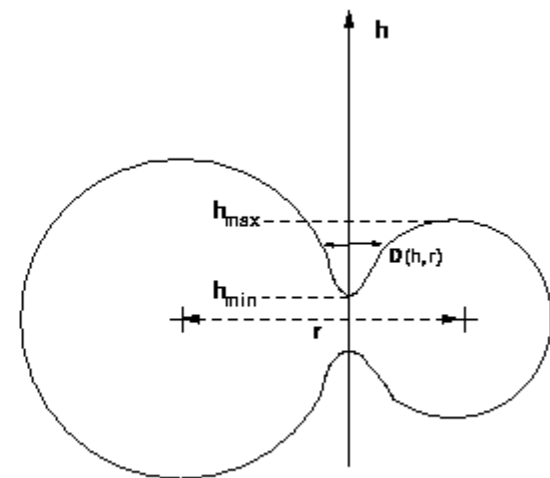
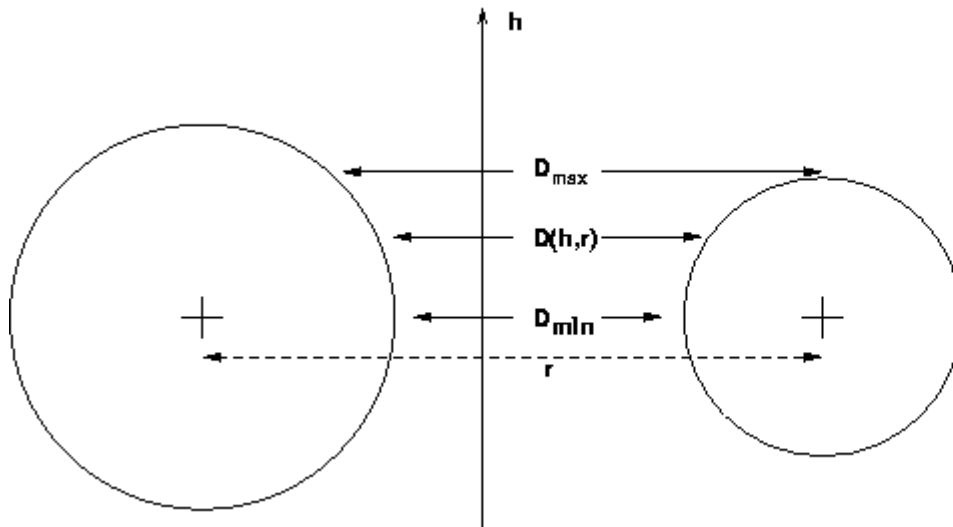
$\theta$ : deviation from the sphere

(G.R., B. Remaud, Nucl. Phys. A 444 (1985) 477)

# Proximity energy

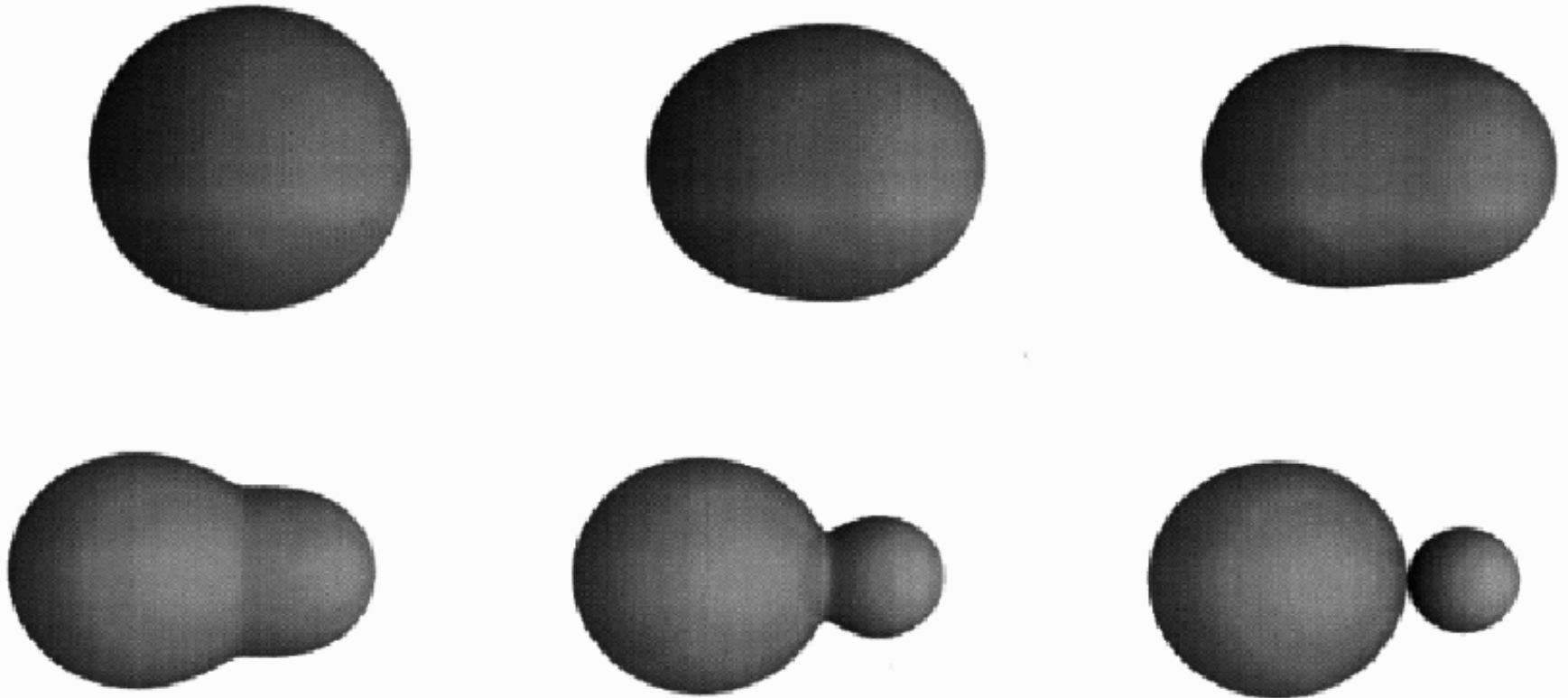
Additional energy to the surface energy taking into account the finite range of the nuclear interaction between opposite nucleons in a gap between incoming nuclei or in a neck in one-body compact shapes ( $\sim -30-40 \text{ MeV}$  at the contact point)

$$E_{\text{proximity}}(r) = 2\gamma \int_{h_{\min}}^{h_{\max}} \phi[D(r, h) / b] 2\pi h dh$$



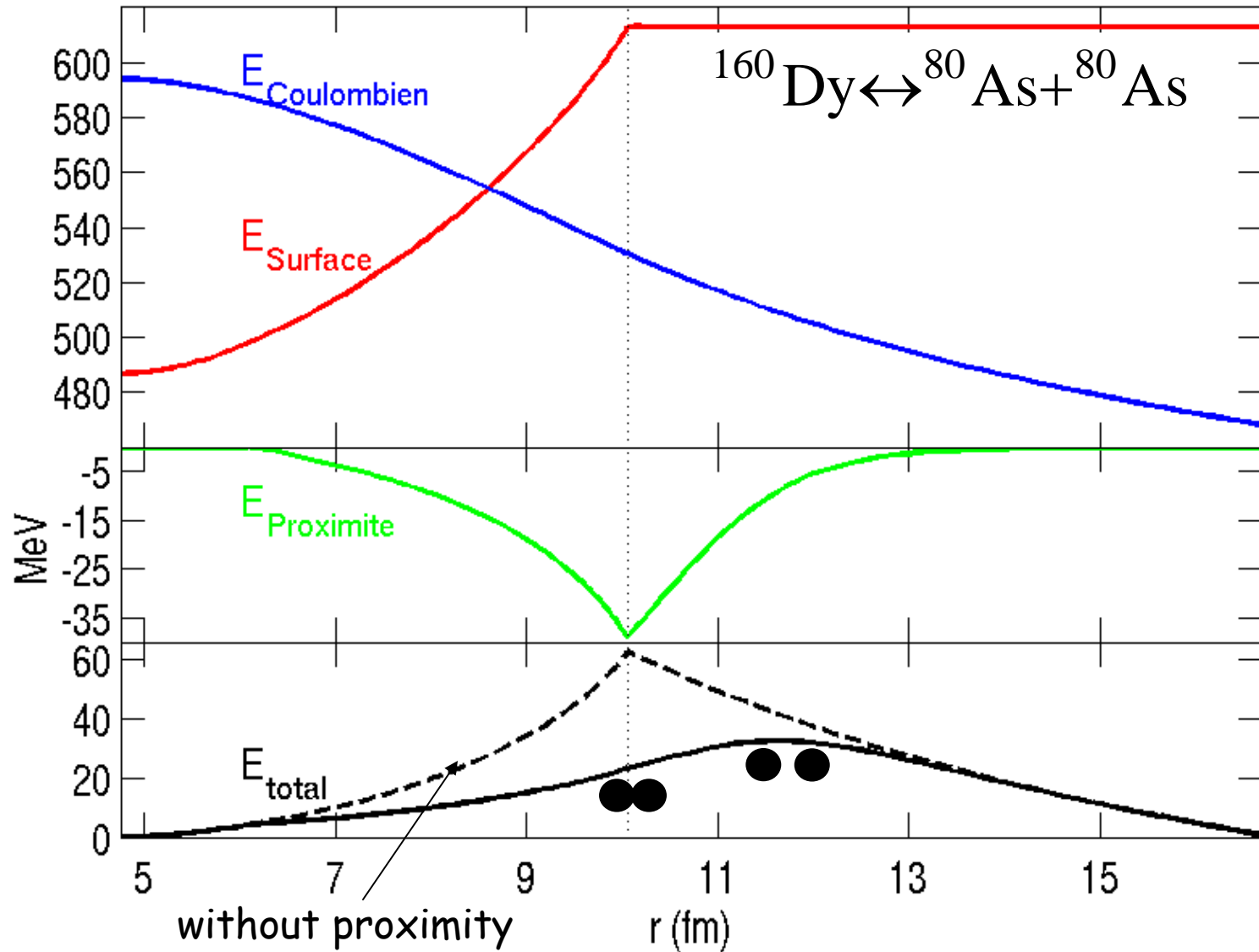
# Elliptic Lemniscatoids

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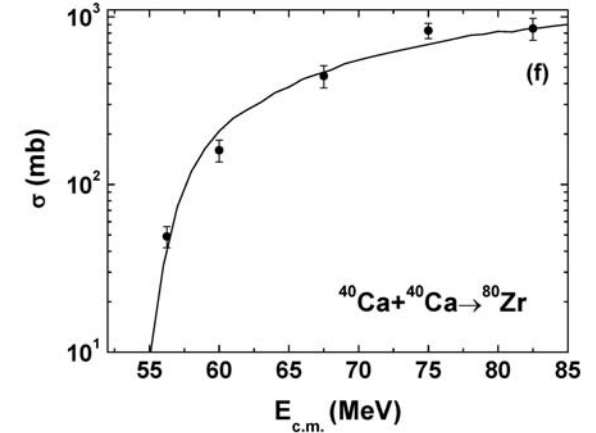
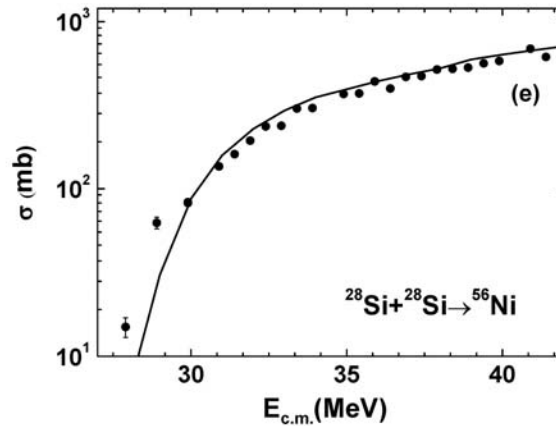
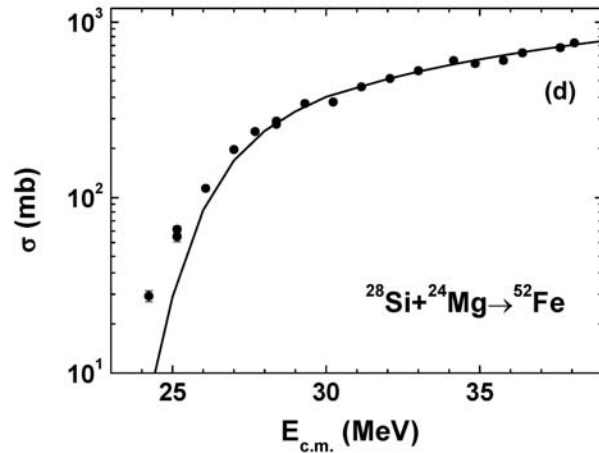
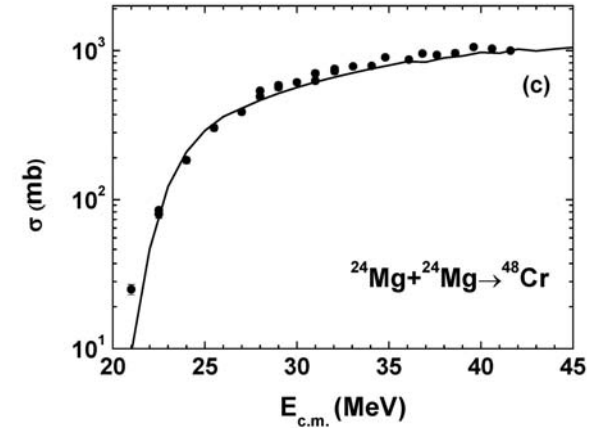
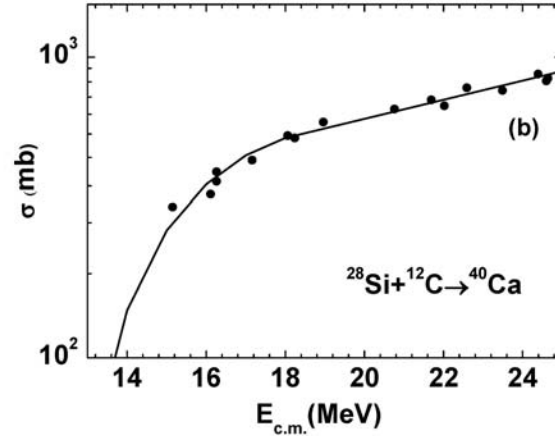
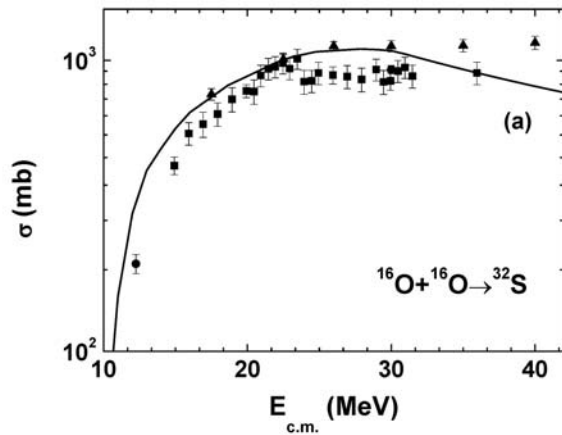


(G.R., B. Remaud, J. Phys. G 8 (1982) L159)

# Evolution of the energy components

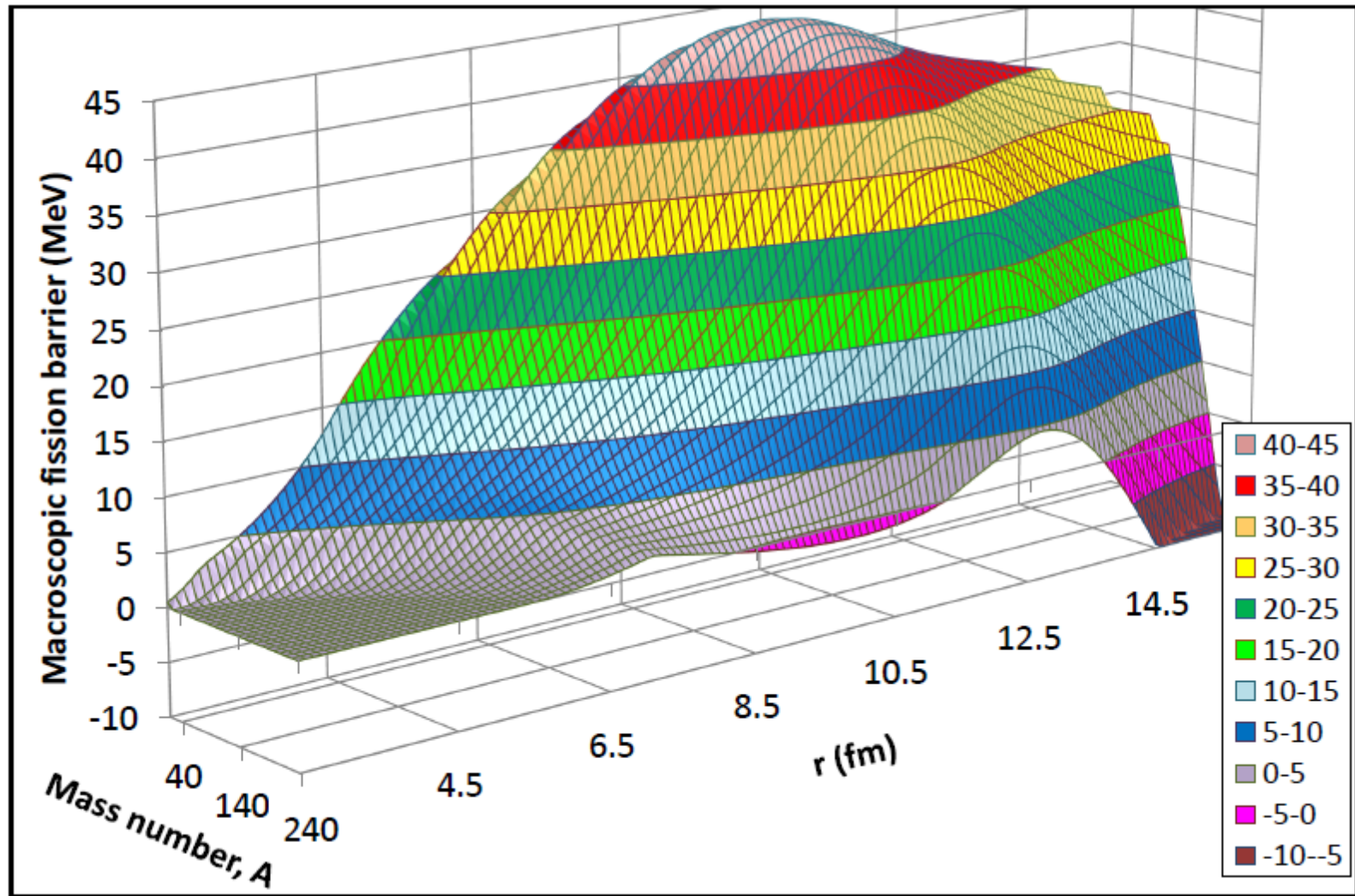


# Fusion cross sections



For light nuclei the friction at the top of the barrier is weak and the fission probability is very low. (G. R., J. Gaudillot, *Phys. Rev. C* 84 (2011) 044602)

# Macroscopic barriers of the symmetric fission of $\beta$ -stable nuclei

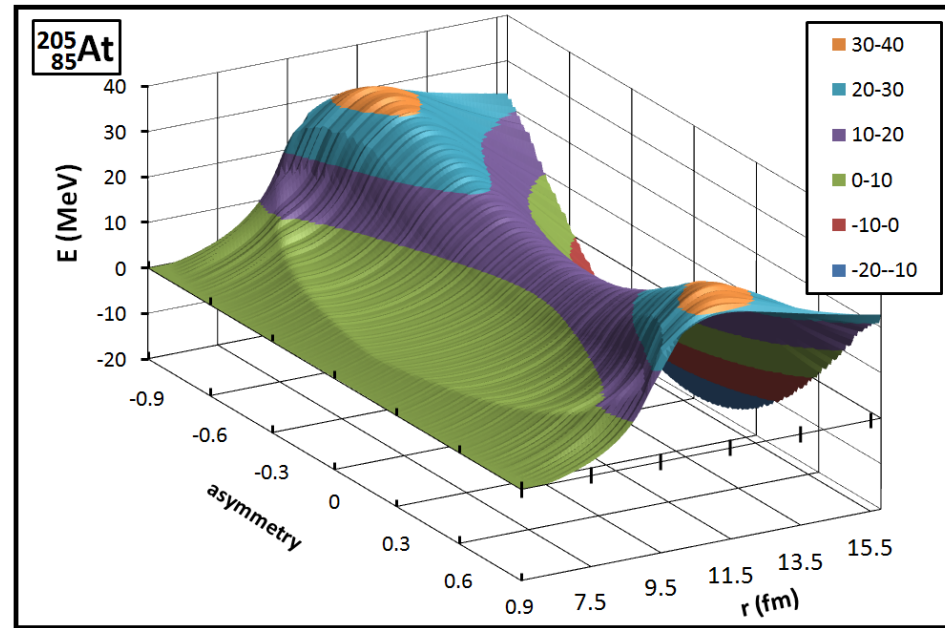
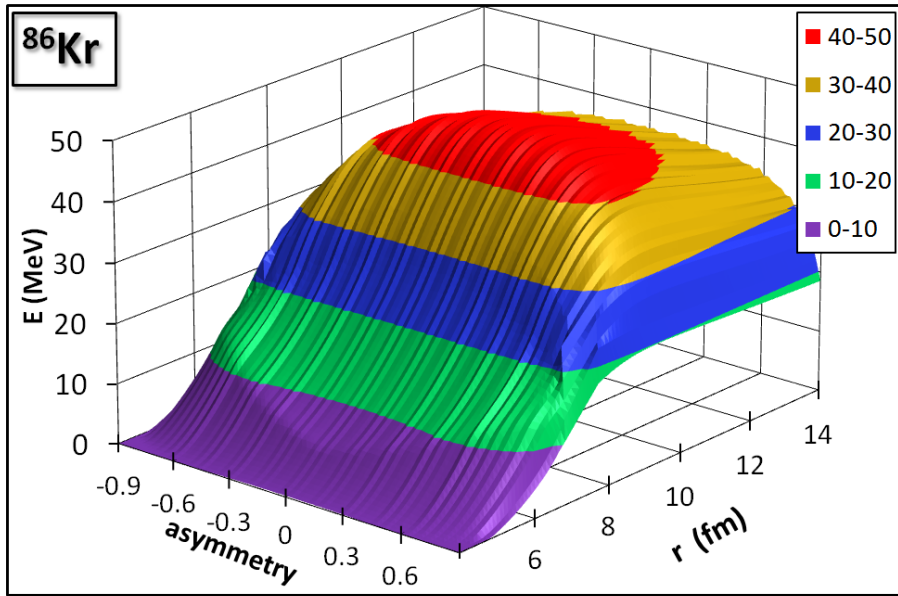


(G.R., M. Jaffré, D. Moreau, Phys. Rev. C 86 (2012) 044326;

X.J. Bao, H.F. Zhang, G.R., J.Q. Li, Nucl. Phys. A 906 (2013) 1)



# Asymmetric fission barriers for $^{86}\text{Kr}$ and $^{205}\text{At}$

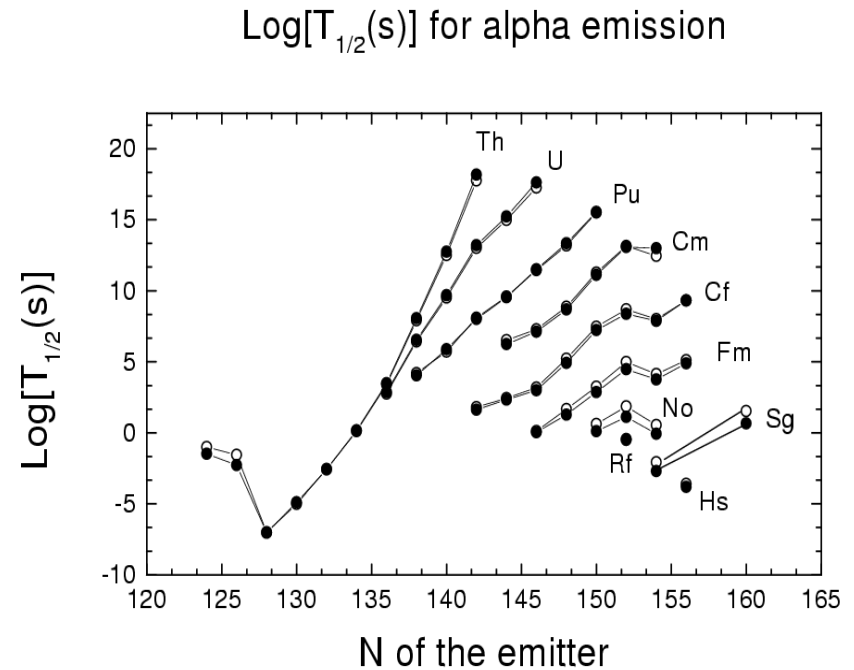
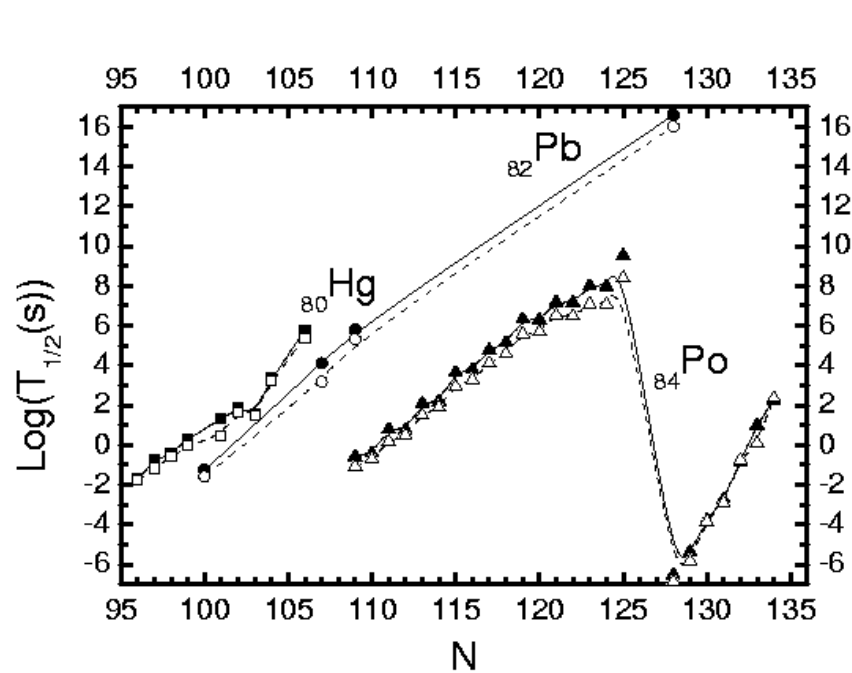


Reproduction of the Businaro-Gallone picture :  
Asymmetric fission for light nuclei and  
symmetric fission for heavy elements  
(macroscopic approach, the shell effects are  
not taken into account)

# Spontaneous fission half-lives of actinides

Reaction	$T_{1/2, \text{exp}(s)}$	$T_{1/2, \text{theo}(s)}$
${}_{92}^{232}\text{U} \rightarrow {}_{52}^{134}\text{Te} + {}_{40}^{98}\text{Zr}$	$2.5 \times 10^{21}$	$4.8 \times 10^{18}$
${}_{92}^{234}\text{U} \rightarrow {}_{52}^{134}\text{Te} + {}_{40}^{100}\text{Zr}$	$4.7 \times 10^{23}$	$4.6 \times 10^{19}$
${}_{92}^{235}\text{U} \rightarrow {}_{50}^{131}\text{Sn} + {}_{42}^{104}\text{Mo}$	$3.1 \times 10^{26}$	$1.1 \times 10^{24}$
${}_{92}^{236}\text{U} \rightarrow {}_{50}^{130}\text{Sn} + {}_{42}^{106}\text{Mo}$	$7.8 \times 10^{23}$	$1.7 \times 10^{22}$
${}_{92}^{238}\text{U} \rightarrow {}_{50}^{130}\text{Sn} + {}_{42}^{108}\text{Mo}$	$2.6 \times 10^{23}$	$5.2 \times 10^{23}$
${}_{94}^{238}\text{Pu} \rightarrow {}_{52}^{132}\text{Te} + {}_{42}^{106}\text{Mo}$	$1.5 \times 10^{18}$	$2.1 \times 10^{18}$
${}_{94}^{239}\text{Pu} \rightarrow {}_{50}^{130}\text{Sn} + {}_{44}^{109}\text{Ru}$	$2.5 \times 10^{23}$	$9.9 \times 10^{22}$
${}_{94}^{240}\text{Pu} \rightarrow {}_{50}^{128}\text{Sn} + {}_{44}^{112}\text{Ru}$	$3.7 \times 10^{18}$	$2.3 \times 10^{20}$
${}_{95}^{243}\text{Am} \rightarrow {}_{51}^{133}\text{Sb} + {}_{44}^{110}\text{Ru}$	$6.3 \times 10^{21}$	$3.6 \times 10^{22}$
${}_{96}^{243}\text{Cm} \rightarrow {}_{48}^{122}\text{Cd} + {}_{48}^{121}\text{Cd}$	$1.7 \times 10^{19}$	$2.3 \times 10^{16}$
${}_{96}^{245}\text{Cm} \rightarrow {}_{50}^{130}\text{Sn} + {}_{46}^{115}\text{Pd}$	$4.4 \times 10^{19}$	$2.0 \times 10^{20}$
${}_{96}^{248}\text{Cm} \rightarrow {}_{50}^{130}\text{Sn} + {}_{46}^{118}\text{Pd}$	$1.3 \times 10^{14}$	$1.9 \times 10^{18}$
${}_{98}^{250}\text{Cf} \rightarrow {}_{49}^{125}\text{In} + {}_{49}^{125}\text{In}$	$5.2 \times 10^{11}$	$4.2 \times 10^{11}$
${}_{99}^{255}\text{Es} \rightarrow {}_{50}^{128}\text{Sn} + {}_{49}^{127}\text{In}$	$8.4 \times 10^{10}$	$5.5 \times 10^6$
${}_{100}^{256}\text{Fm} \rightarrow {}_{50}^{128}\text{Sn} + {}_{50}^{128}\text{Sn}$	$1.0 \times 10^4$	$1.1 \times 10^4$
${}_{102}^{256}\text{No} \rightarrow {}_{51}^{128}\text{Sb} + {}_{51}^{128}\text{Sb}$	110	$1.9 \times 10^0$

# Experimental and theoretical alpha decay half-lives



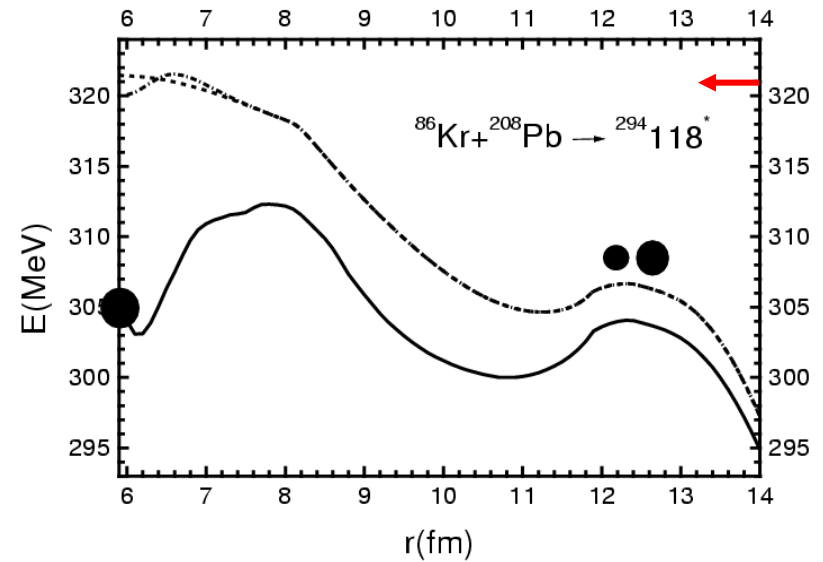
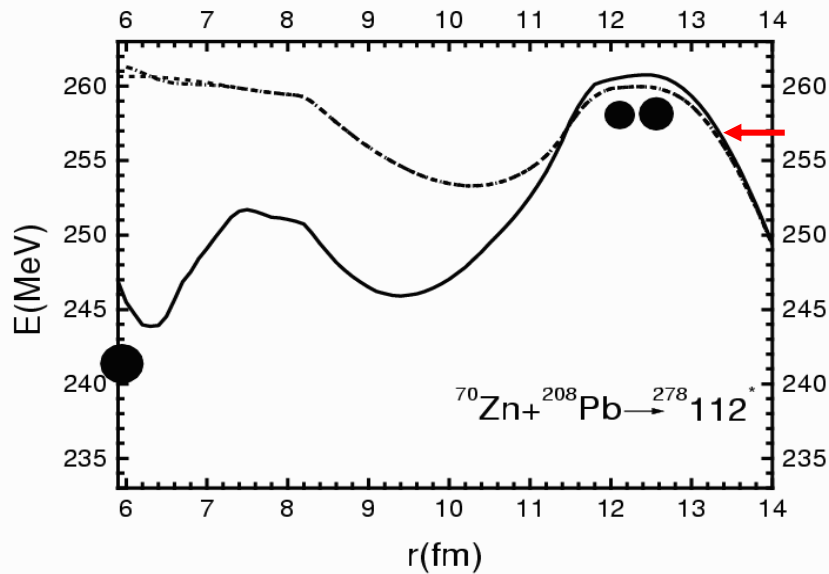
$\alpha$  decay half-lives around  $^{208}\text{Pb}$  and for the heaviest elements

rms deviation between the theoretical (LDM) and experimental values :  
0.35 for 131 e-e nuclei and 0.63 for the whole set of 373 alpha emitters.

(G. R., J. Phys. G 26 (2000) 1149)

# Cold fusion barriers for the heaviest nuclei

## Potential barriers in the $^{70}\text{Zn}$ and $^{86}\text{Kr}$ on $^{208}\text{Pb}$ reactions

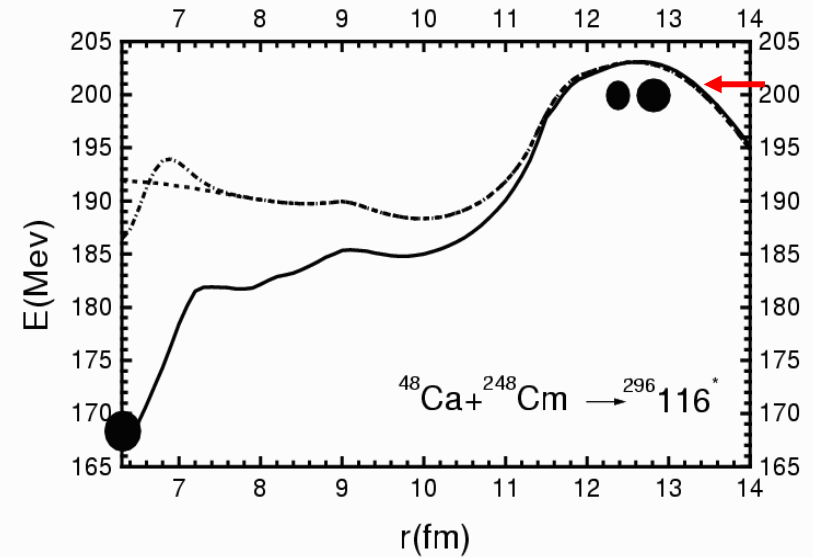
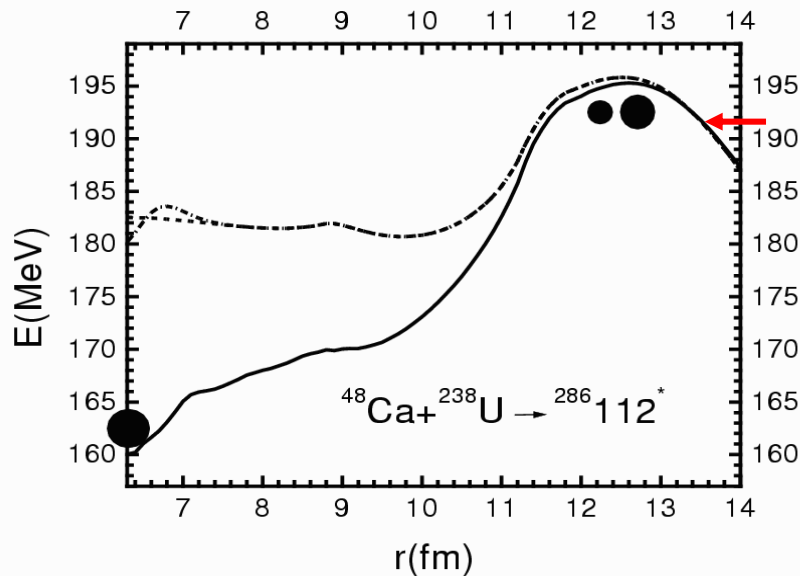


Double-humped fusion barriers  
Possible fast fission events  
Low excitation energy

(G.R., R.A. Gherghescu, Nucl. Phys. A  
699 (2002) 479)

# Warm fusion barriers for the heaviest nuclei

## Potential barriers in the $^{48}\text{Ca}$ on $^{238}\text{U}$ and $^{248}\text{Cm}$ reactions



High excitation energy

No deep double-humped fusion barriers

No fast fission event in a second well

# Recent Data : Z = 117

( $^{48}\text{Ca}+^{249}\text{Bk}$  reaction, Oganessian et al, PRL 104 (April 2010) 142502)

Comparison between the experimental and theoretical half-lives

A	293	289	285	294	290	286	282	278	274
Z	117	115	113	117	115	113	111	109	107
$T_{\text{exp}}$	10-25 ms	0.14- 0.48 s	3.7- 10.5 s	0.042- 0.45 s	0.016 s	19.6 s	0.51 s	7.6 s	54 s
$T_{\text{The}}$	9.7-24 ms	0.15- 0.54 s	3.1-12 s	0.15- 0.54 s	1.18- 323 s	16.7- 71.3 s	314- 1513 s	0.48- 7.1 s	41-194 s
$Q_{\text{exp}}$ (MeV)	11.18 +-0.08	10.45 +-0.1	9.75 +-0.1	10.96 +-0.1	10.09 +-0.4	9.76 +-0.1	9.13 +-0.1	9.69 +-0.19	8.93 +-0.1

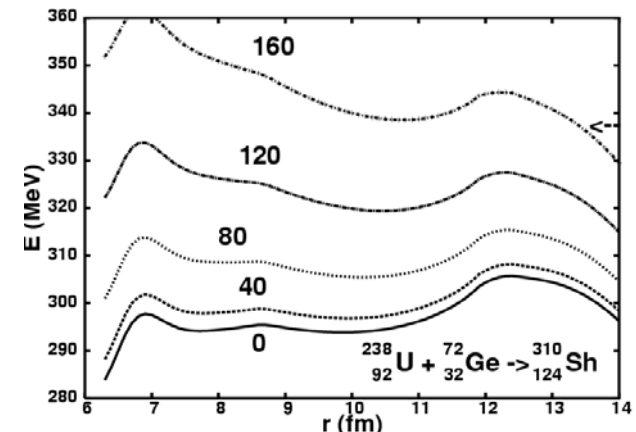
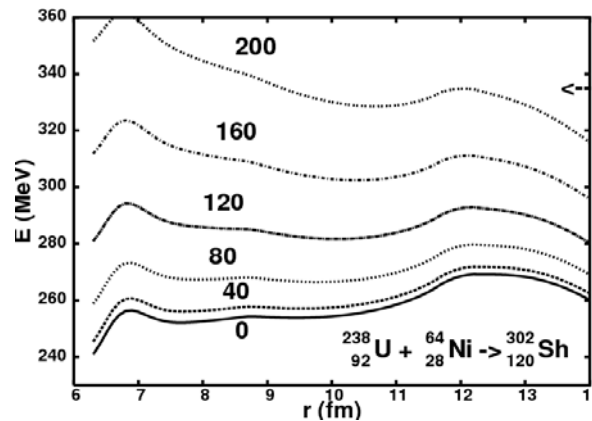
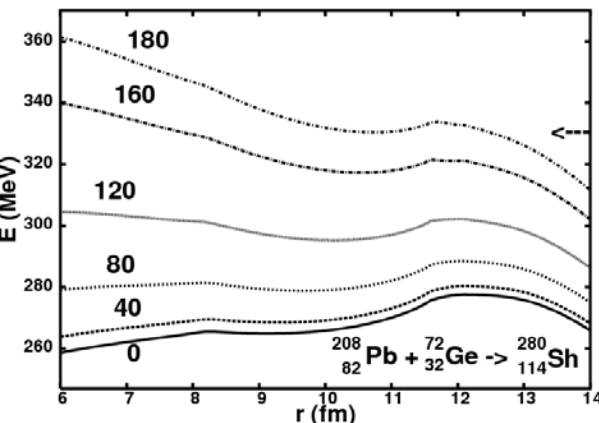
# Superheavy elements formed in the Pb+Ge, U+Ni and U+Ge reactions ?

Experiments performed at GANIL in reverse kinematics at high excitation energy : resp. 71, 95 and 55 MeV for Pb+Ge (Z=114), U+Ni (Z=120) and U+Ge (Z=124).

Analyses of the associated reaction time distributions provide evidence for nuclei with Z=120 and 124 living longer than  $10^{-18}$ s arising from highly excited compound nuclei.

(Exp : M. Morjean et al, Phys. Rev. Lett 101, 072701 (2008); Phys. Rev. Lett 108, 122701 (2012),...).

**Fusion barriers as a function of the angular momentum :** ( $\leftarrow$  gives the beam energy).



Theoretically, the excitation energy being very large, very high angular momenta are populated and the potential barrier is very flat allowing **the possible formation of rapidly rotating isomeric states without reaching a real compound nucleus formation** and even though the shell effects vanish ? (G. R., Phys. Rev. C 87 (2013) 057601)

# Liquid Drop Model mass formula

$$B = a_v(1 - k_v I^2)A - a_s(1 - k_s I^2)A^{2/3} - 0.6e^2 Z^2 / R_0 + a_0 A^0 \\ - a_k(1 - k_k I^2)A^{1/3} + f_p Z^2 / A + a_{c,exc} Z^{4/3} / A^{1/3} - E_{pair} - E_{shell} - E_{Wigner}$$

The pairing and shell effects of the **Thomas-Fermi model** of Myers and Swiatecki (Nucl. Phys. A 601 (1996) 141) have been used.

$$(E_{Wigner} = W|I|, \quad c_1 \exp(-c_2 I^2), \quad c|N - Z| \exp[-(A/50)^2], \quad c|N - Z| \exp[-(A/35)]) \\ \text{or } E_{cong} = -10 \exp(-4.2|I|) \text{ MeV} \quad (\text{S. Goriely et al, PRC 66 (2002) 024326})$$

The coefficients are adjusted on 2027 masses ( $N, Z > 7, \quad \sigma < 150 \text{ keV}$ ),  
(2003 Atomic Mass Evaluation, Nucl. Phys. A 729( 2003) 337)



# Main Results

The basic terms in  $A$ ,  $AI^2$ ,  $A^{2/3}$ ,  $A^{2/3}I^2$  and  $Z^2/A^{1/3}$  lead to

$\sigma = 0.95 \text{ MeV}$ .

-The Coulomb diffuseness correction term or the charge exchange correction term play the main role to improve the accuracy of the mass formula and lead to  $\sigma = 0.63 \text{ MeV}$ .

-The usual Wigner energy term and the curvature energy term can be used independently and are very efficient but their coefficients are very changeable.

The constant and congruence terms are not very efficient.

-The surface coefficient varies between 16.5 and 18.5 MeV.

-  $r_0$  is around 1.21-1.24 fm.

(G. R., M. Guilbaud, A. Onillon, Nucl. Phys. A 847 (2010) 24,  
G.R., A. Subercaze, Nucl. Phys. A 917 (2013) 1)

# Conclusion

- A Generalized Liquid Drop Model including the mass and charge asymmetries, the proximity energy, the rotational energy, the shell and pairing energies has been used to describe the transition between two-body and one-body shapes in entrance and exit channels of nuclear reactions.
- In the quasimolecular shape valley where the proximity energy is maximal at the contact point, double-humped potential barriers begin to appear even macroscopically for heavy systems in entrance and exit channels due to the influence of the proximity forces and, consequently, quasimolecular isomeric states can survive in the second minimum of the potential barriers.
- The calculated  $l$ -dependent fusion and fission barrier heights and half-lives of alpha decay and actinides agree with the experimental data.
- The liquid drop model allows also to investigate the nuclear mass formulas.
- Analytical formulas have been proposed to calculate the fusion, fission and alpha and cluster radioactivity data and nuclear masses.
- Thanks to :
  - C. Beck (Strasbourg), V.Yu Denisov (Kyiv), R. Gherghescu (Bucarest), W. von Oertzen (Berlin); H.F. Zhang, J.Q. Li, W. Zuo (Lanzhou).
  - From Nantes : C. Bonilla, F. Haddad, A. Onillon, M. Jaffré, D. Moreau, J. Gaudillot, A. Subercaze.