(International Symposium on Physics of Unstable Nuclei 2014, Ho Chi Minh City)

^{8}Be, ^{12}C, ^{16}O,.. nuclei and alpha clustering within a Generalized Liquid Drop Model

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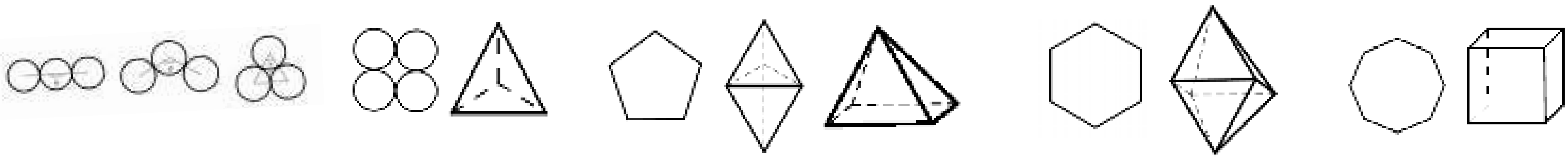
- Generalized Liquid Drop Model
- Proximity energy
- Applications in fission, fusion and alpha emission
- N-alphas : ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{32}S



Liquid Drop Model energy

$$E_{GLDM}=E_{vol}+E_{surface}+E_{Coulomb}+E_{proximity}+E_{shell}+E_{pairing}$$

$$E_{vol}=-a_v(1-k_vI^2)A$$

$$E_{surf}=a_s(1-k_sI^2)A^{2/3}\times\frac{Surf}{4\pi R_0^2}$$

$$E_{Coul}=\frac{9e^2Z^2}{16\pi^2R_0^6}\iiint\frac{d\tau d\tau'}{\left|\vec{r}-\vec{r}'\right|}$$

$$E_{shell}=E_{shell}^{sphere}(1-3.1\theta^2)e^{-\theta^2}$$

$$E_{pairing}=E_{TF\ model}$$

$I=(N-Z)/A$

$a_v=15.494\ \mathrm{MeV}$

$a_s=17.9439\ \mathrm{MeV}$

$k_v=1.8$

$k_s=2.6$

$R_0=1.28A^{1/3}-0.76+0.8A^{-1/3}$

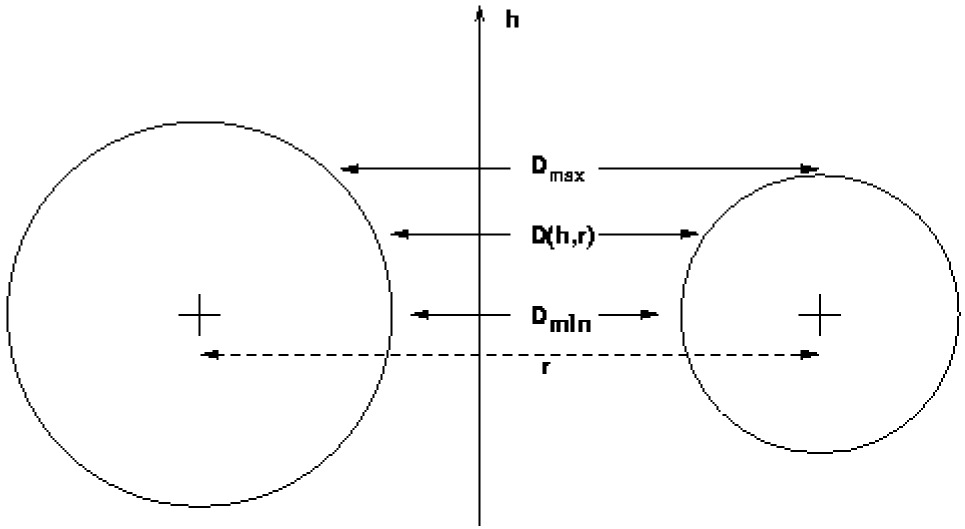
θ: deviation from the sphere

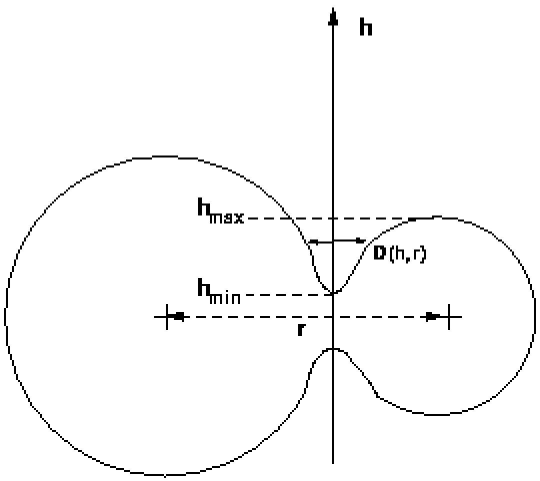
(G.R., B. Remaud, Nucl. Phys. A 444 (1985) 477)

Proximity energy

Additional energy to the surface energy taking into account the finite range of the nuclear interaction between opposite nucleons in a gap between incoming nuclei or in a neck in one-body compact shapes (~ - 35 MeV at the contact point for heavy nuclei and 9.4 MeV for two alphas)

$$E_{proximity}(r) = 2\gamma \int_{h_{min}}^{h_{max}} \phi[D(r,h)/b] 2\pi h dh$$

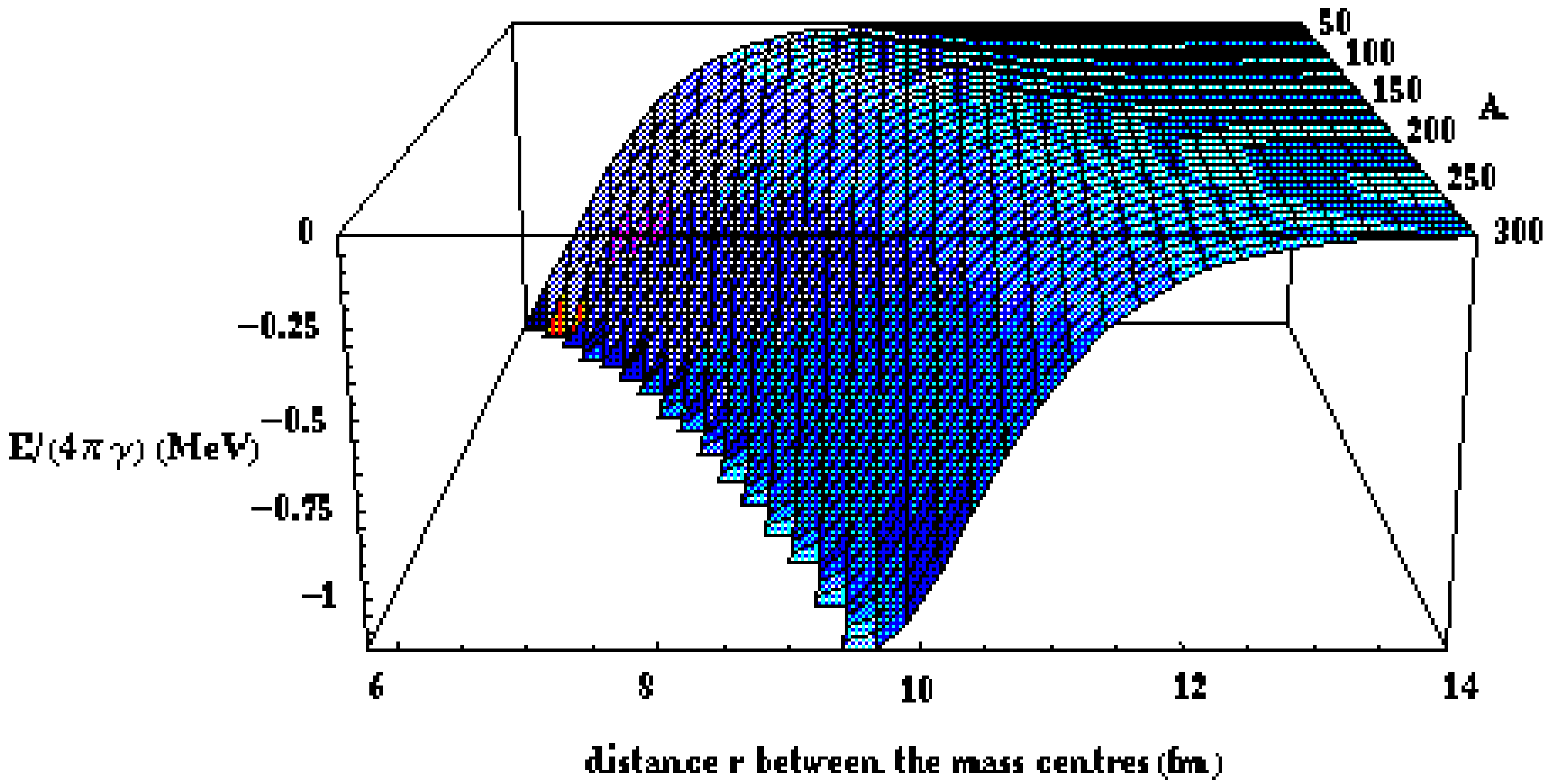




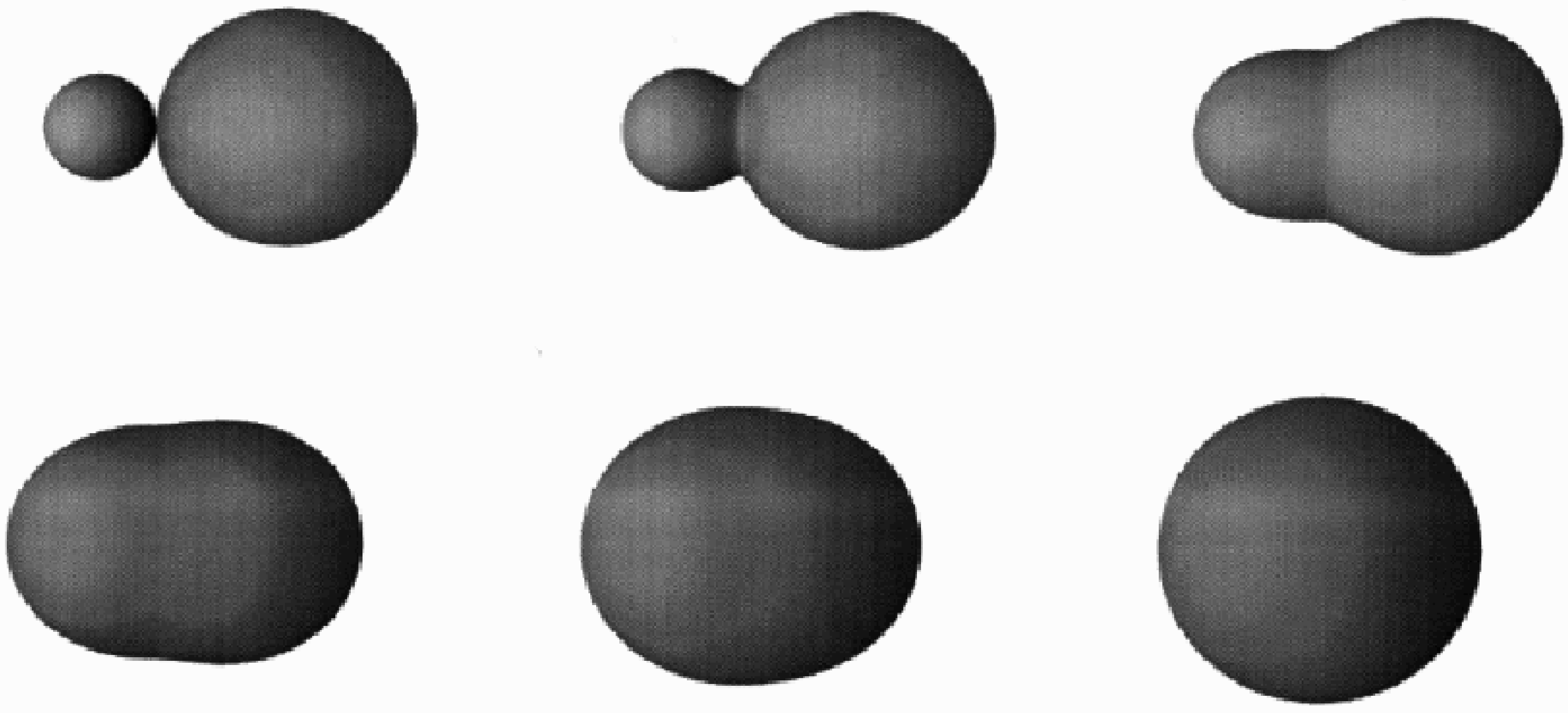
Analytical proximity energy for the alpha emission

$E_{Proximity}$ from the contact point between an alpha particle and the daughter nucleus :

$$\frac{E_{pr}}{4\pi\gamma} = e^{-1.38(r-R_{cont})}\left[0.6584A^{2/3} - \left(\frac{0.172}{A^{1/3}} + 0.4692A^{1/3}\right)r - 0.02548A^{1/3}r^2 + 0.01762r^3\right]$$

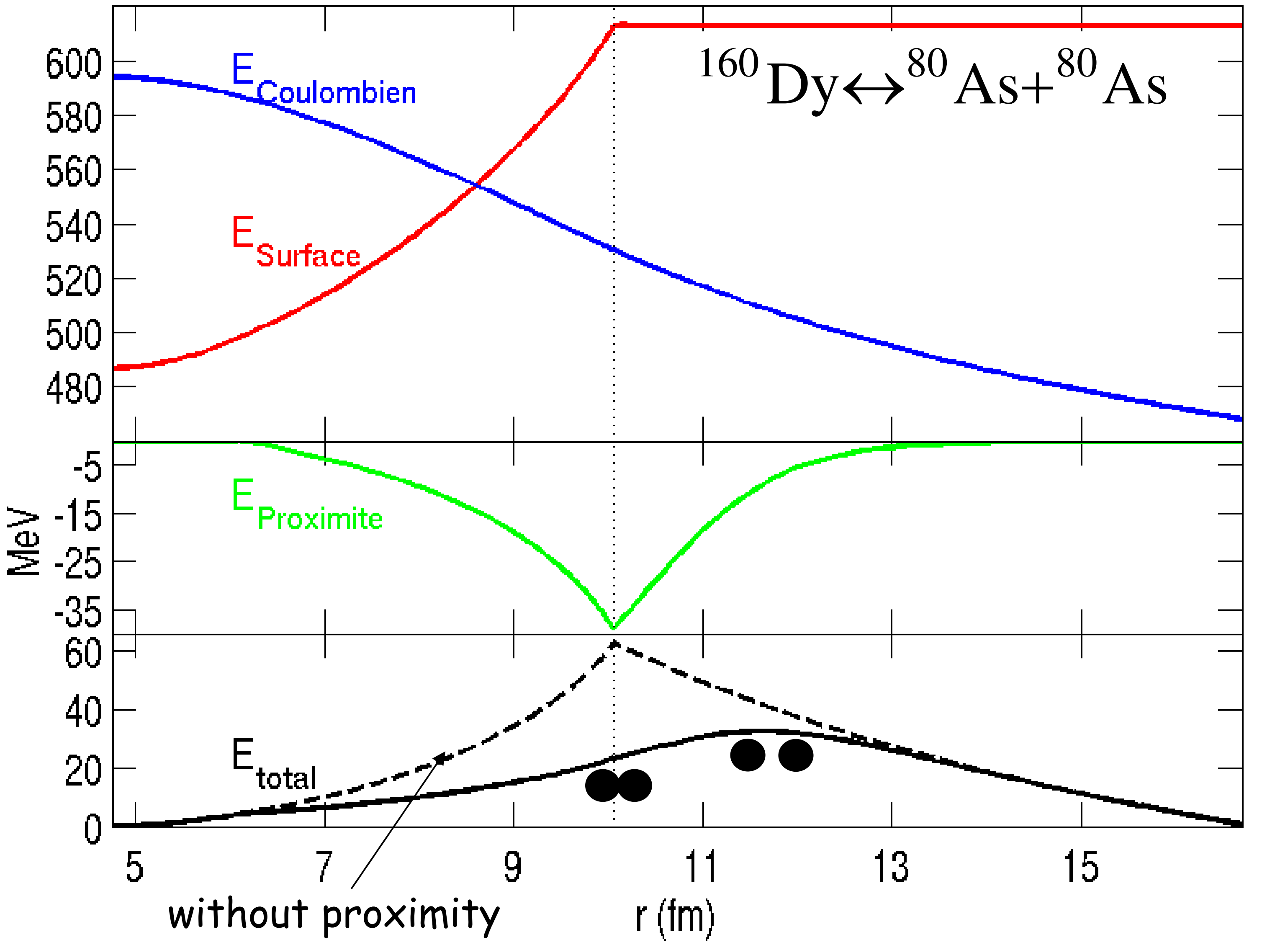


Elliptic Lemniscatoïds

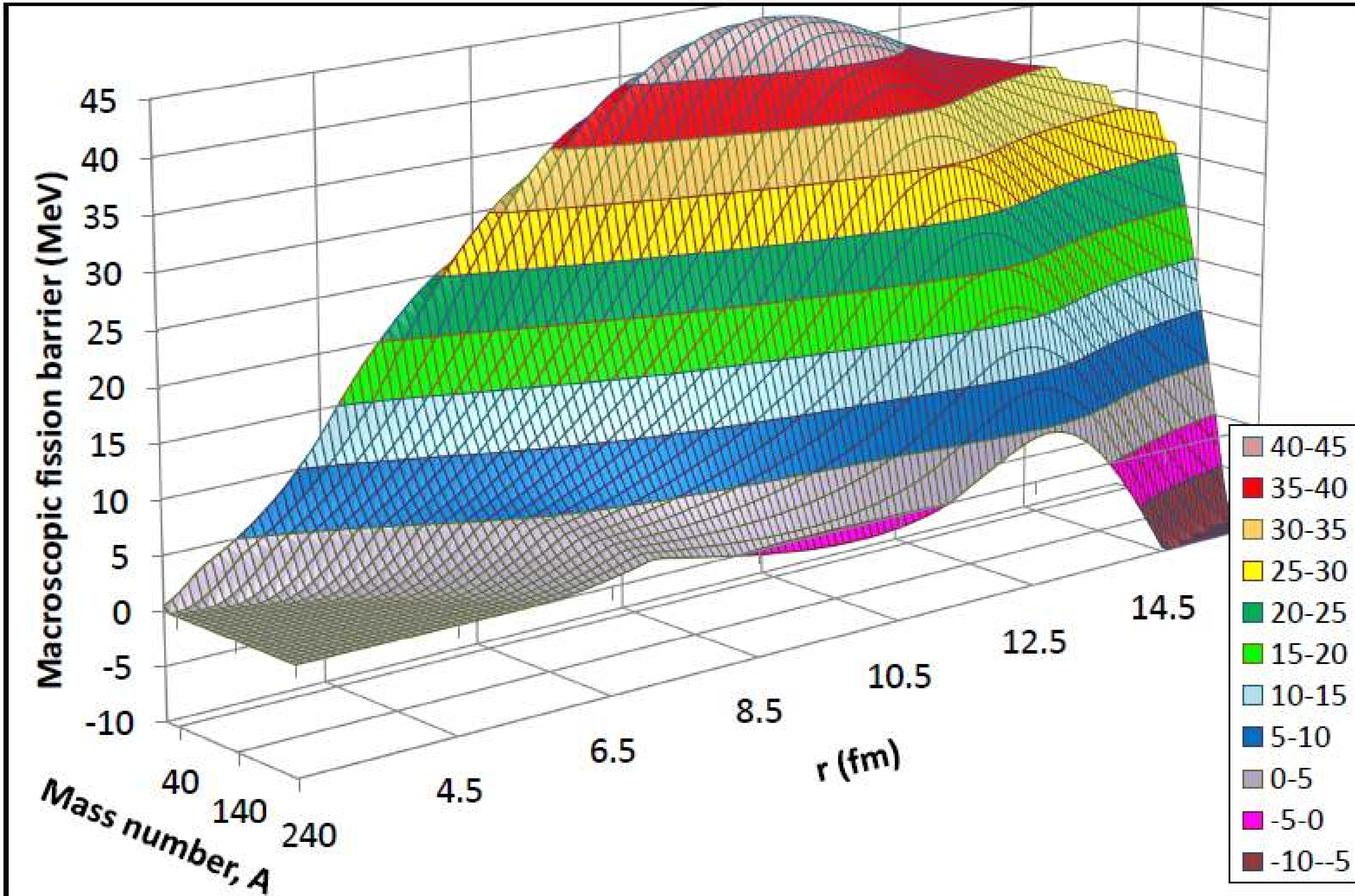


(G.R., B. Remaud, J. Phys. G 8 (1982) L159)

Evolution of the macroscopic energy components

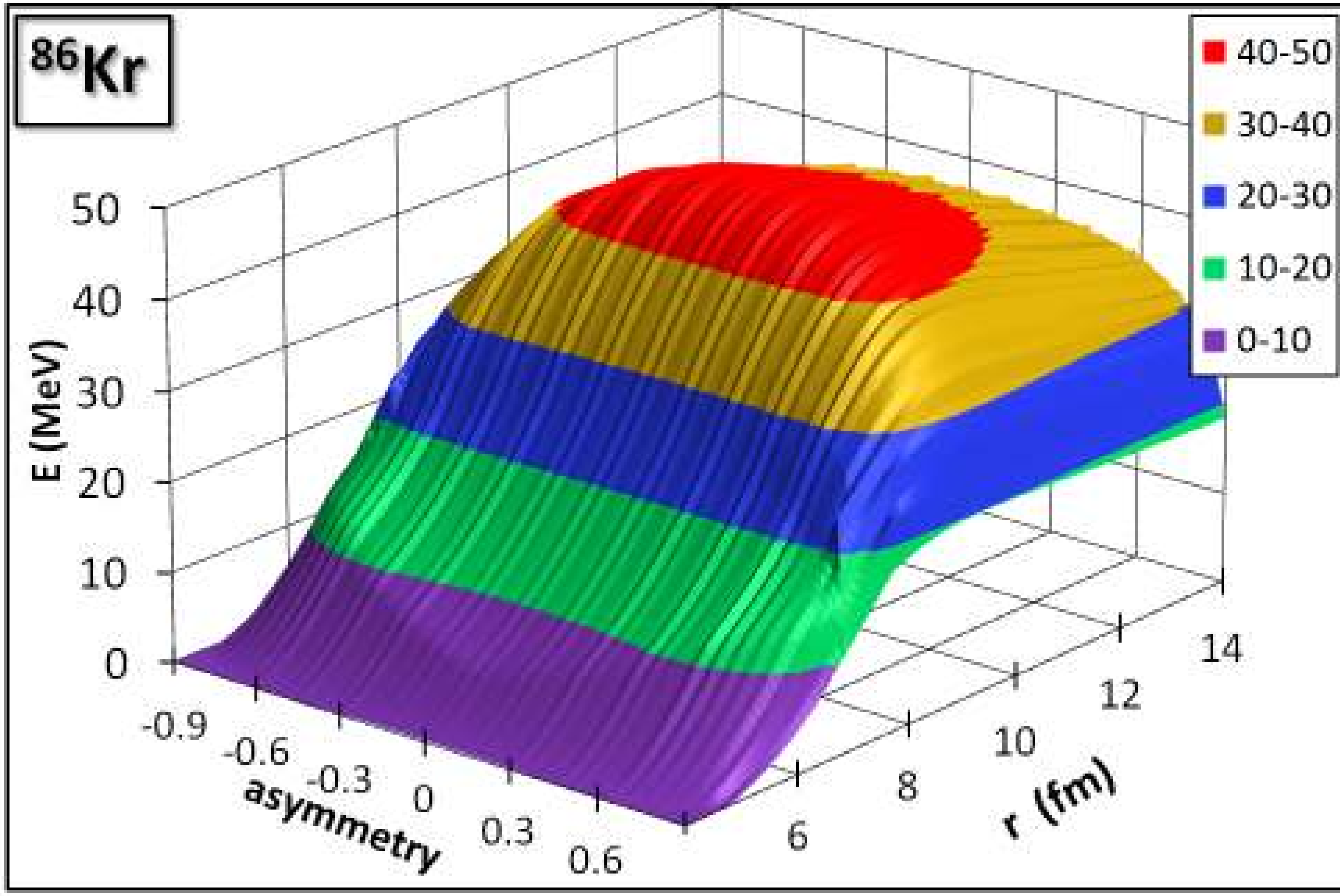


Macroscopic barriers of the symmetric fission of β-stable nuclei

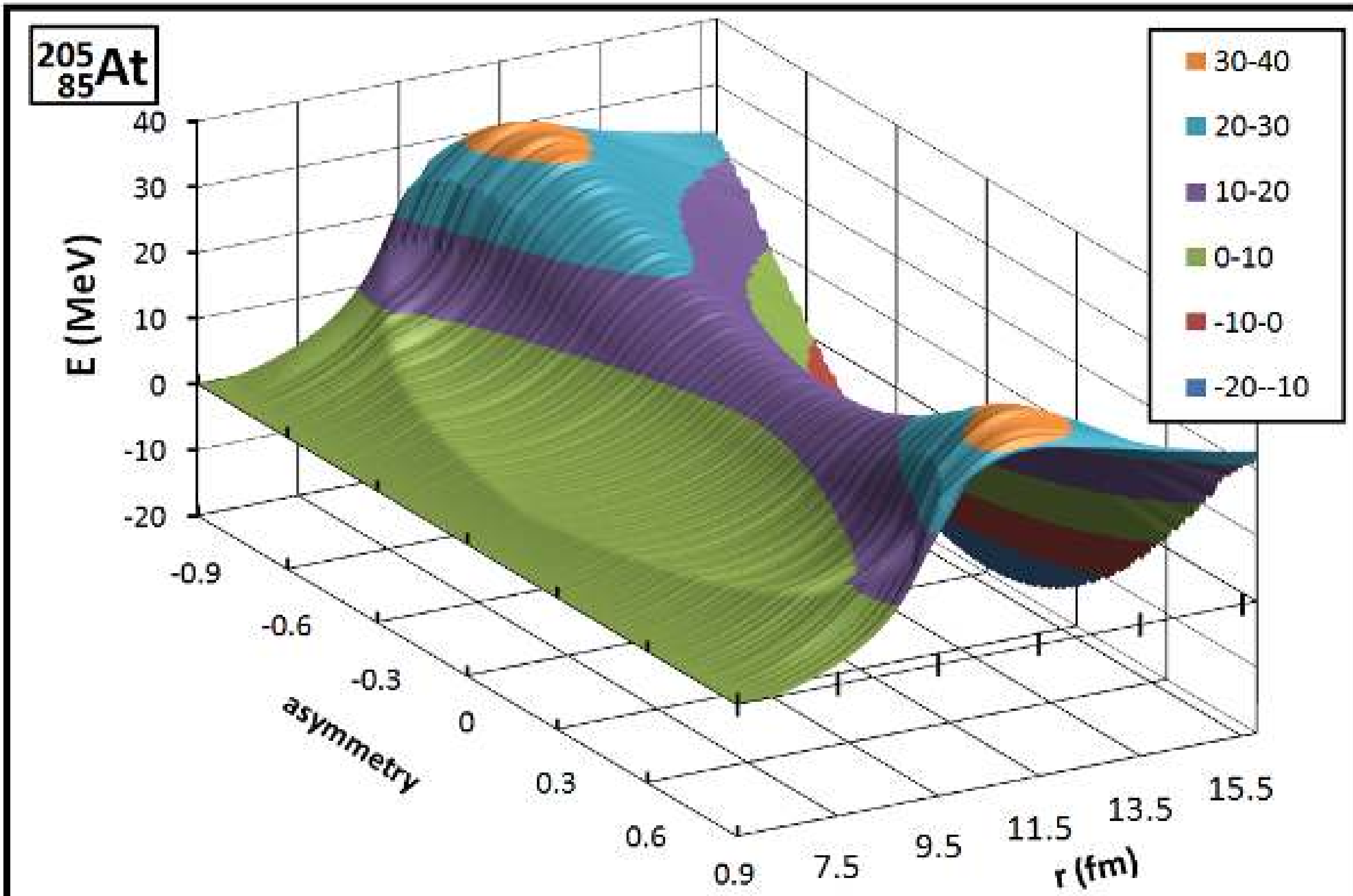


(G.R., M. Jaffré, D. Moreau, Phys. Rev. C 86 (2012) 044326)

Asymmetric fission barriers for ^{86}Kr



Asymmetric fission barriers for ^{205}At

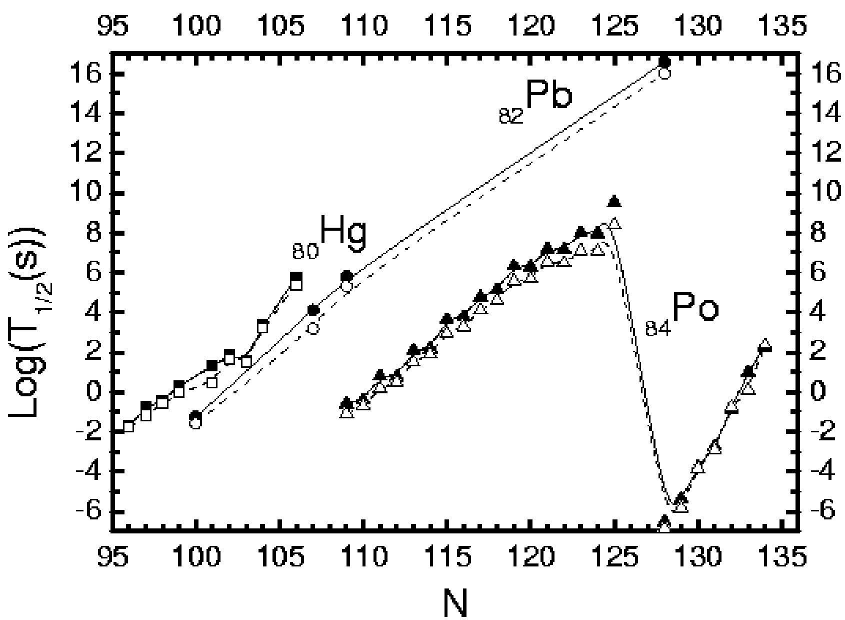


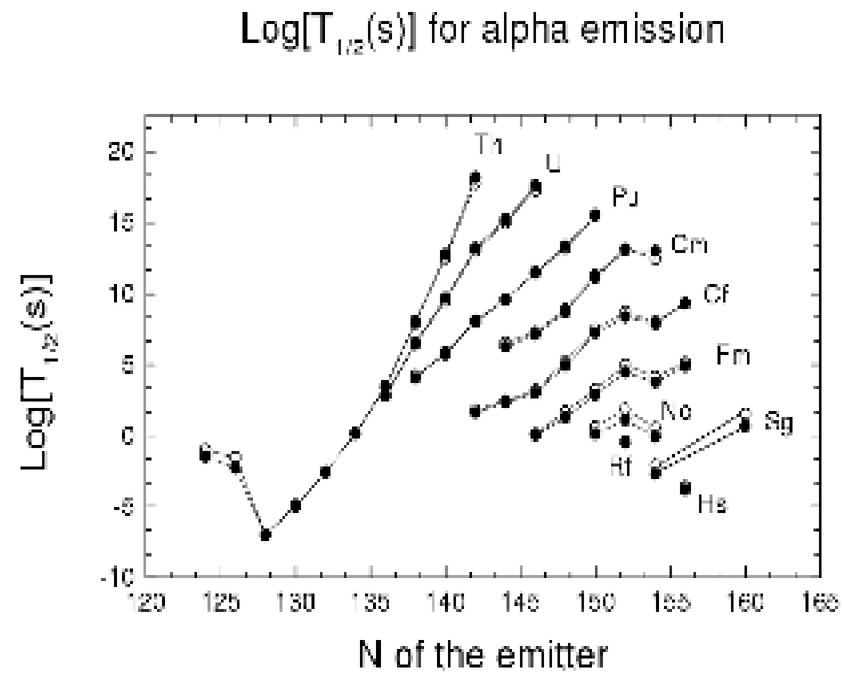
Fusion barrier heights and positions

Reaction	E_0 (exp.) (MeV)	R_0(exp.) (fm)	E_0 (GLDM) (MeV)	R_0 (GLDM) (fm)
$^{9}Be+^{10}B$	3.4	7.65	3.36	7.79
$^{4}He+^{44}Ca$	6.37	8.25	6.20	8.51
$^{4}He+^{59}Co$	8.26	8.63	8.10	8.83
$^{4}He+^{164}Dy$	17.14	10.32	16.98	10.45
$^{4}He+^{209}Bi$	20.52	10.88	20.46	10.93
$^{4}He+^{233}U$	21.69	11.45	22.23	11.16
$^{40}Ca+^{58}Ni$	73.36	10.20	74.72	9.98

(G. Royer, J. Phys. G 12 (1986) 623)

Experimental and theoretical alpha decay half-lives





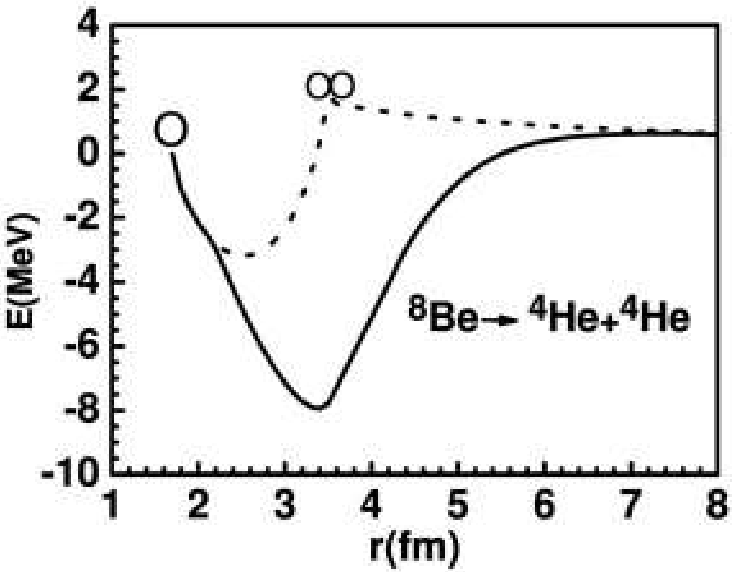
α decay half-lives around ^{208}Pb and for the heaviest elements

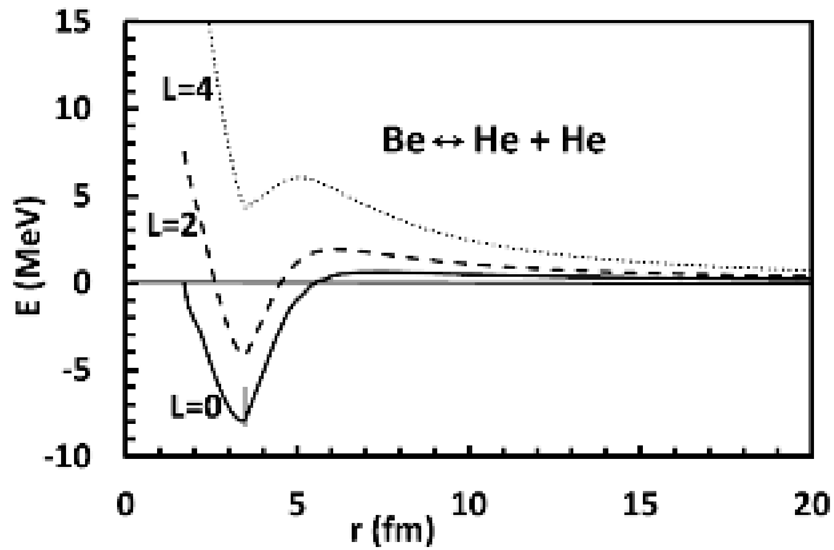
rms deviation between the theoretical (LDM) and experimental values :

0.35 for 131 e-e nuclei and 0.63 for the whole set of 373 alpha emitters.

(G. R., J. Phys. G 26 (2000) 1149)

^{8}Be nucleus ($T_{1/2}$ = 8.2 10^{-17} s)





L-dependent potential barriers for the ^{8}Be <-> $^{4}He+^{4}He$ reaction.

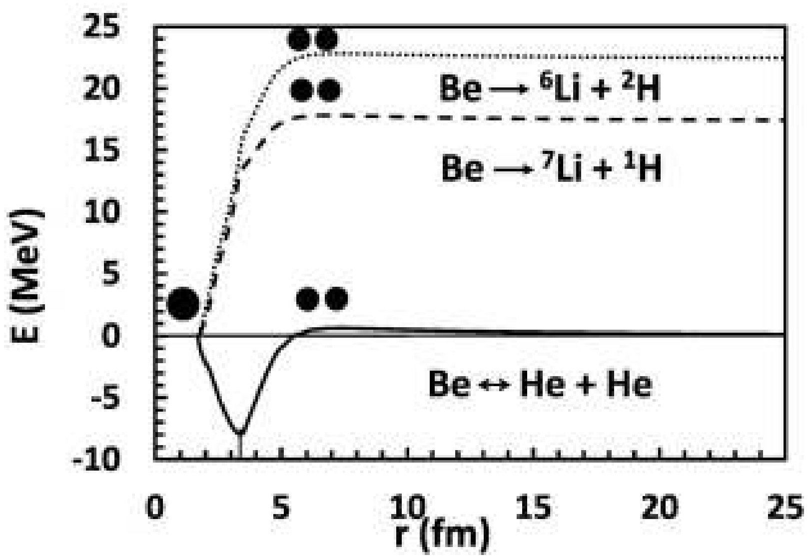
$$E_{\text{rotation}} = \frac{\hbar^2 l(l+1)}{2 I_{\perp}}$$

Deformation energies calculated without (---) and with (__) a proximity energy term.

Energies of the 2^+ and 4^+ states : 3.03 and 11.35 MeV

Theoretical energies of the 2 and 4 states : 3.78 and 12.25 MeV.

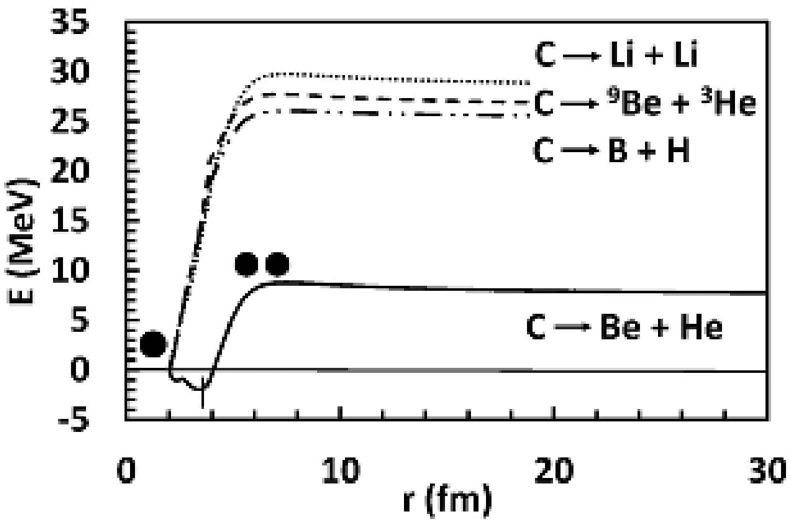
^{8}Be nucleus



Potential barriers governing the ^{8}Be <-> ^{6}Li+^{2}H ^{8}Be <-> ^{7}Li+^{1}H and ^{8}Be <-> ^{4}He+^{4}He reactions.

	r_{sph}	$r_{E_{min}}$	r_{cont}	$r_{E_{max}}$	∞
$^{8}Be \leftrightarrow {}^{4}He + {}^{4}He$					
r (fm)	1.65	3.39	3.49	7.45	
E (MeV)	0	−7.95	−7.84	0.62	−0.0918
$^{8}Be \rightarrow {}^{7}Li + {}^{1}H$					
r (fm)	1.73		3.33	6.89	
E (MeV)	0		12.94	17.82	17.25
$^{8}Be \rightarrow {}^{6}Li + {}^{2}H$					
r (fm)	1.69		3.42	7.38	
E (MeV)	0		15.43	22.81	22.28

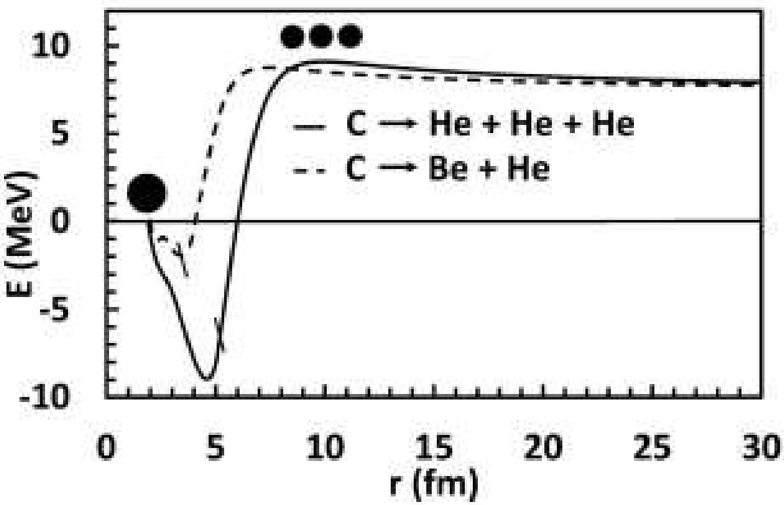
^{12}C nucleus



Potential barriers governing the ^{12}C <-> ^{8}Be+^{4}He, ^{12}C <-> ^{10}B+^{2}H, ^{12}C <-> ^{9}Be+^{3}He and ^{12}C <-> ^{6}Li+^{6}Li reactions.

	r_{sph}	$r_{E_{min}}$	r_{cont}	$r_{E_{max}}$	∞
$^{12}C \leftrightarrow {}^{8}Be + {}^{4}He$					
r (fm)	1.91	3.41	3.96	7.43	
E (MeV)	0.00	−1.95	−0.63	8.77	7.365
$^{12}C \rightarrow {}^{10}B + {}^{2}H$					
r (fm)	1.98		3.81	7.49	
E (MeV)	0.00		18.26	26.06	25.19
$^{12}C \rightarrow {}^{9}Be + {}^{3}He$					
r (fm)	1.94		3.90	7.12	
E (MeV)	0.00		20.97	27.73	26.28
$^{12}C \rightarrow {}^{6}Li + {}^{6}Li$					
r (fm)	1.89		4.0	7.48	
E (MeV)	0.00		19.63	29.44	28.17

^{12}C nucleus

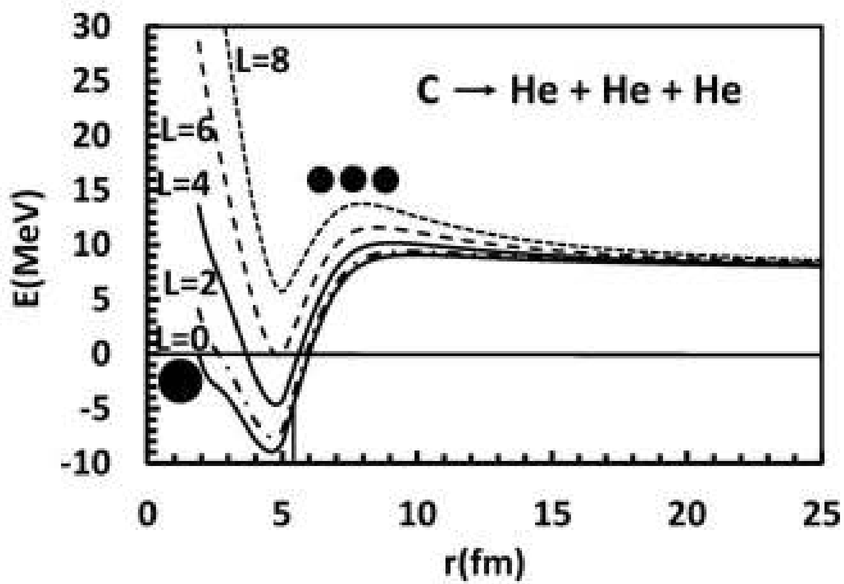


Potential barriers governing the binary $^{12}C \leftrightarrow {}^{8}Be + {}^{4}He$ and prolate ternary $^{12}C \leftrightarrow {}^{4}He + {}^{4}He + {}^{4}He$ reactions.

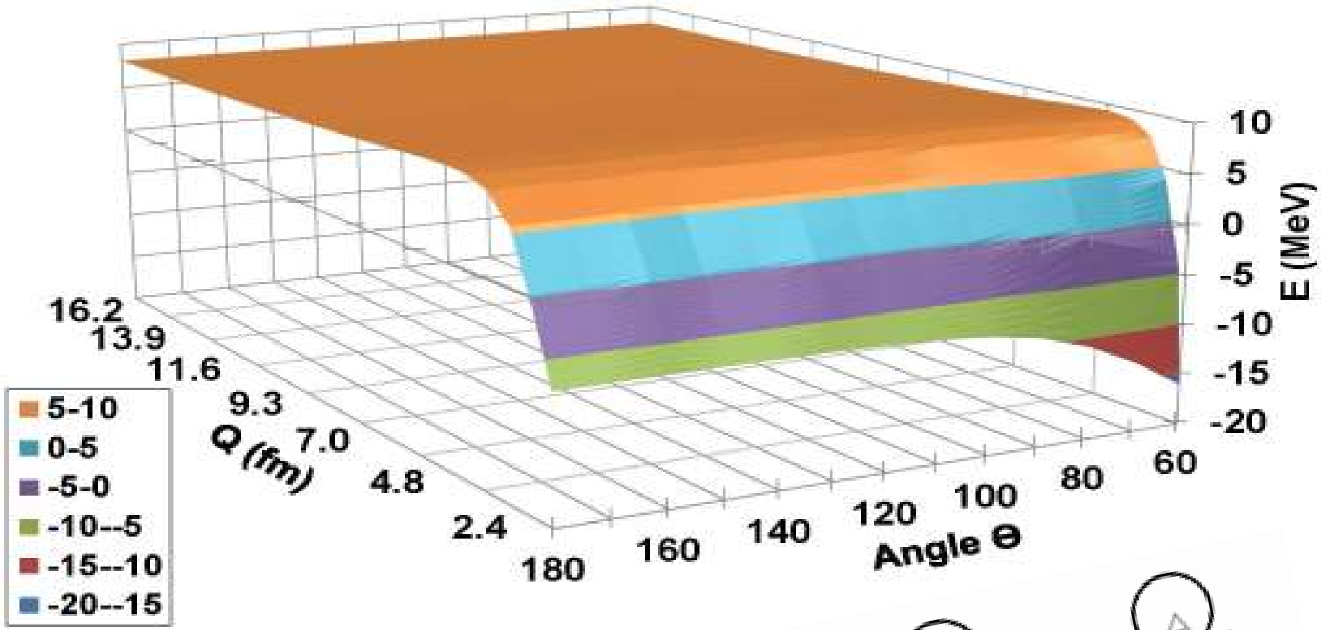
$Q_{3\alpha} = 7.27$ MeV

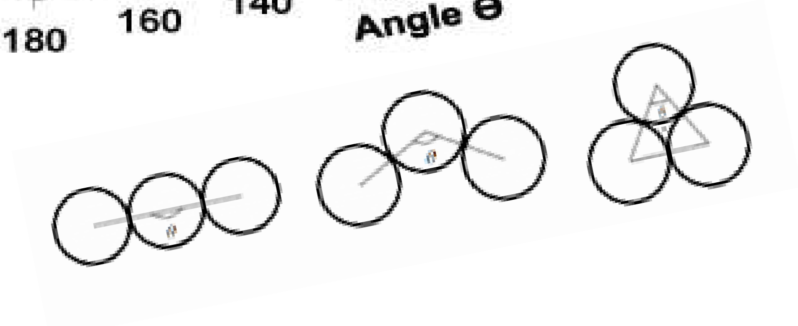
$Q_{Be+He} = 7.37$ MeV

L-dependent barriers for the prolate ternary $^{12}C \leftrightarrow {}^{4}He + {}^{4}He + {}^{4}He$ reaction.



^{12}C nucleus

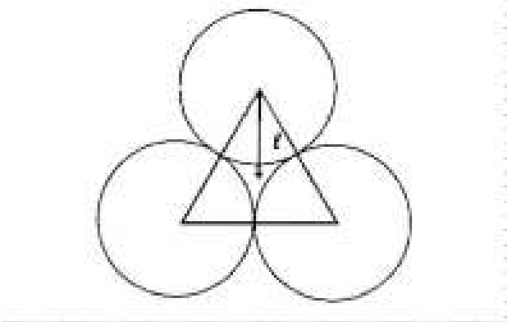


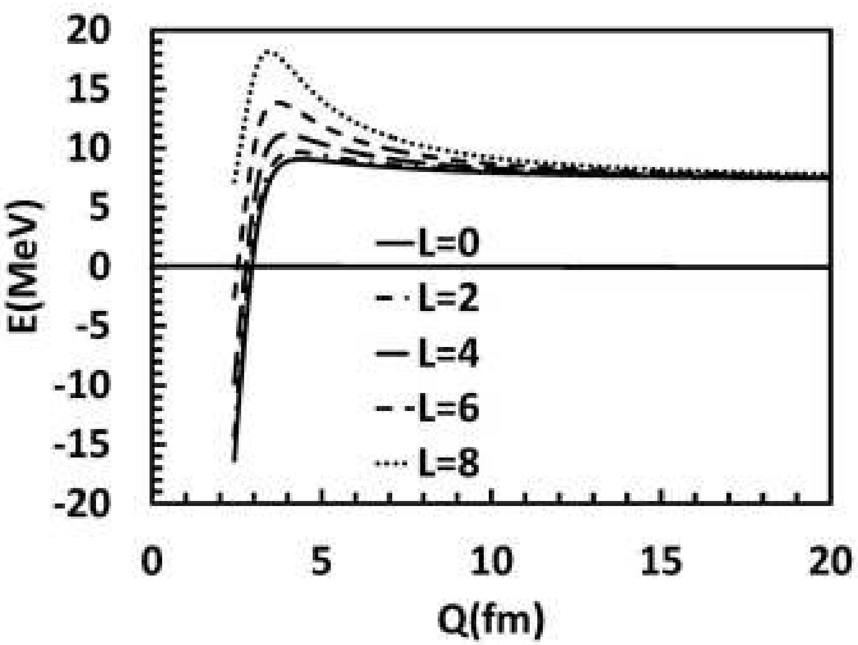


Deformation barrier versus the three-alphas configuration.

^{12}C nucleus

$\langle r^2\rangle = l^2 + 0.6\, R_0^2\, n^{-2/3}$





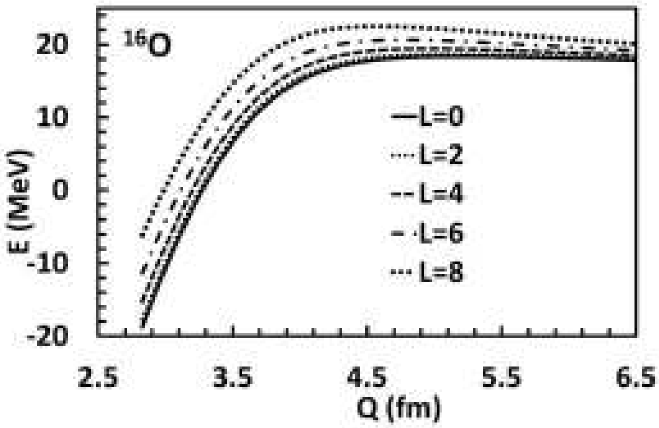
$\langle r^2\rangle^{1/2}$ (exp) = 2.47 fm
$\langle r^2\rangle^{1/2}$ (GLDM) = 2.43 fm
(for a linear chain 3.16 fm)

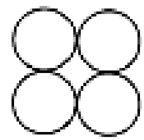
Electric quadrupole moment :
Q_0 (exp) = -22+-10 e fm^2
Q_0 (GLDM) = -24.4 e fm^2

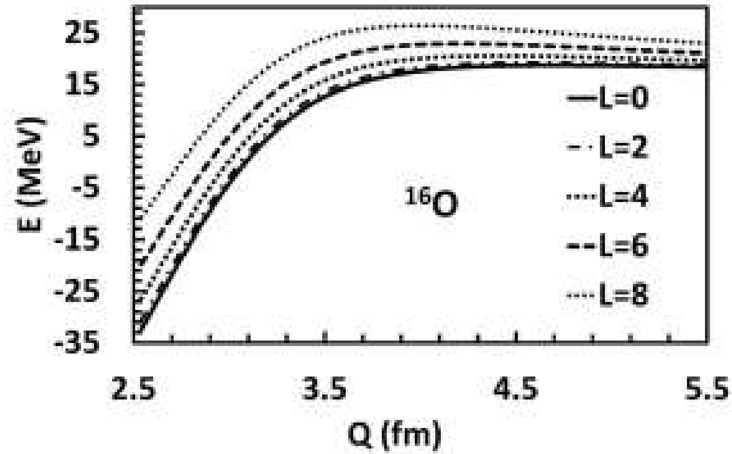
L-dependent barriers for the
$^{12}C \leftrightarrow {}^4He+{}^4He+{}^4He$ triangular configuration.
Q = $\langle r^2\rangle^{1/2}$ is the r.m.s radius.

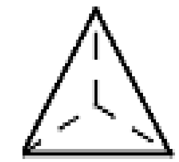
The difference between the energies of the minima of the linear chain configuration and the minima of the oblate equilateral configuration is 7.36 MeV, close to the energy 7.65 MeV of the excited Hoyle state.
(G. R., A. Escudie, B. Sublard, Phys. Rev. C 90 (2014) 024607)

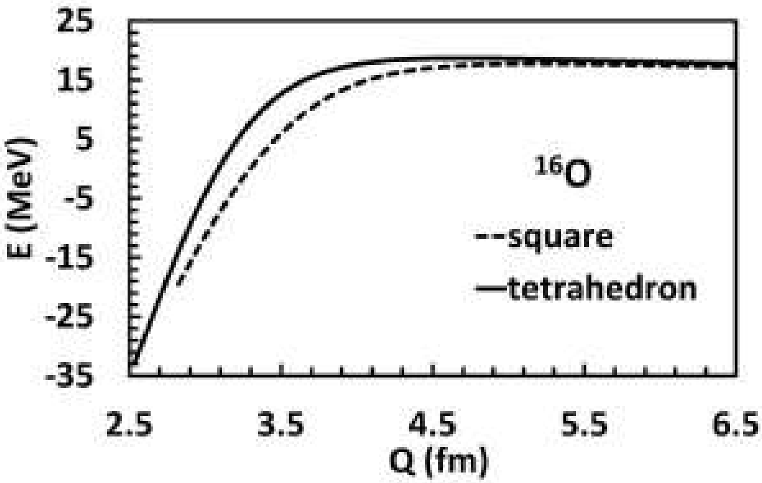
^{16}O nucleus

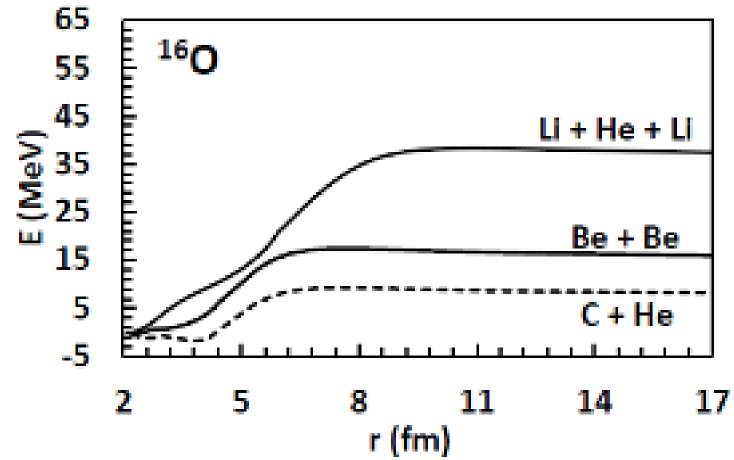








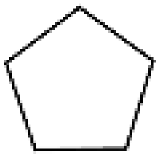


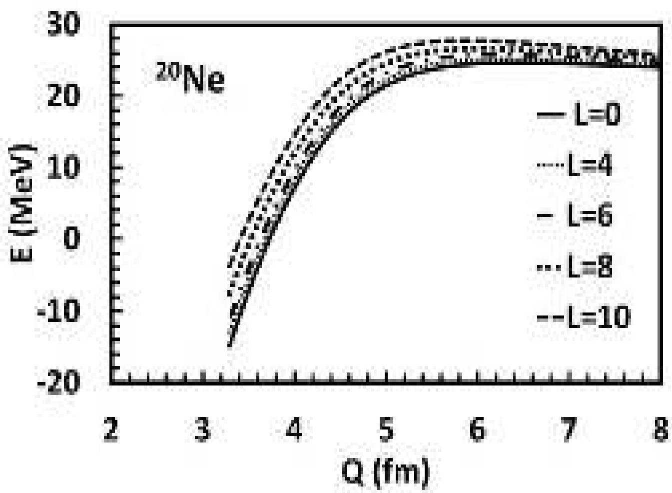


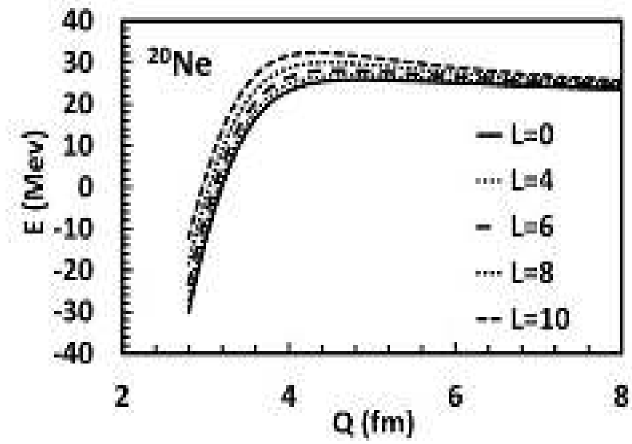
$$Q_{4\alpha} = 14.44 \text{ MeV}$$

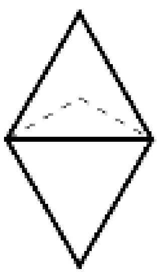
Reaction	$^{12}C+^{4}He$	$^{8}Be+^{8}Be$	$^{6}Li+^{4}He+^{6}Li$
$Q_{reaction}$ (MeV)	7.16	14.62	35.34

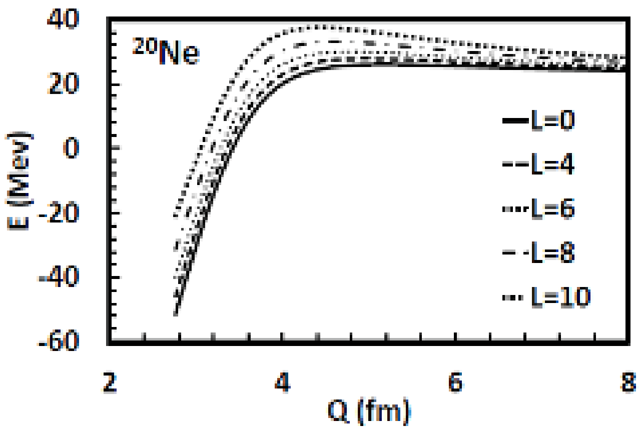
^{20}Ne nucleus

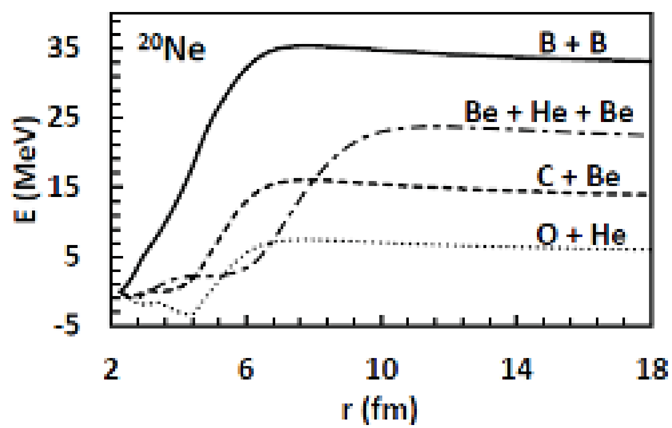








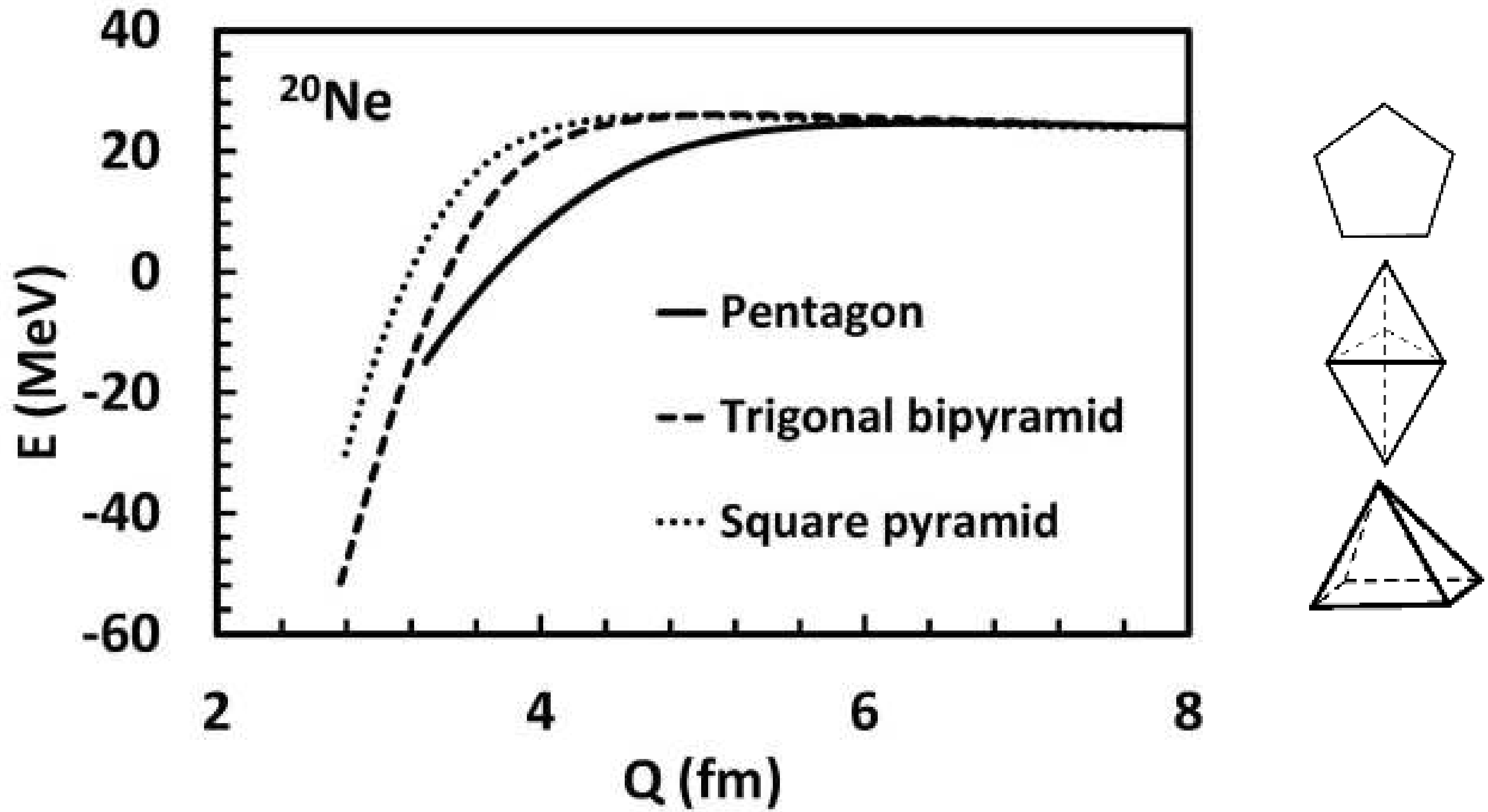




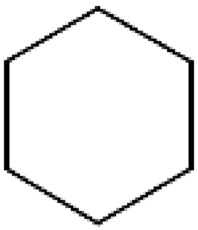
$Q_{5\alpha} = 19.17$ MeV

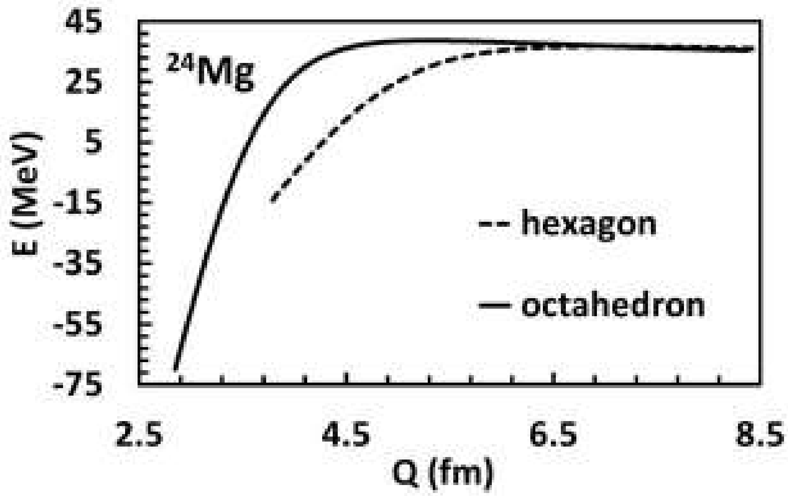
Reaction	$^{16}O+^{4}He$	$^{12}C+^{8}Be$	$^{8}Be+^{4}He+^{8}Be$	$^{10}B+^{10}B$
Q(MeV)	4.73	11.98	19.35	31.14

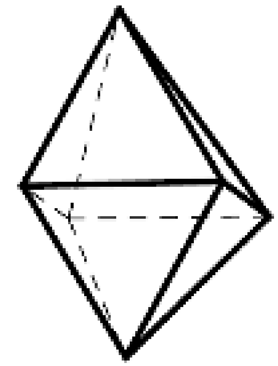
^{20}Ne nucleus

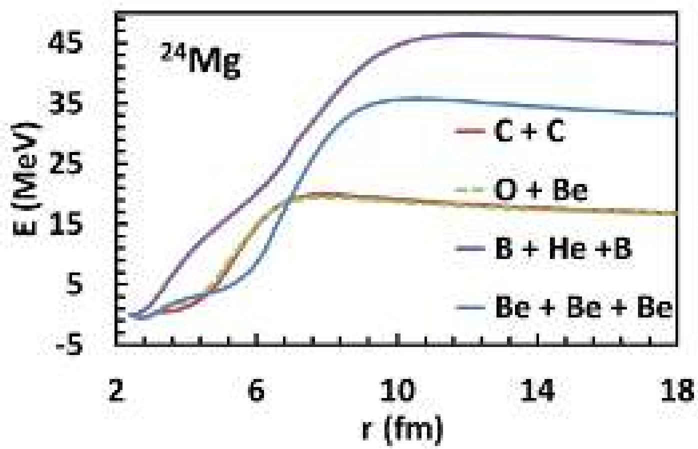


^{24}Mg nucleus





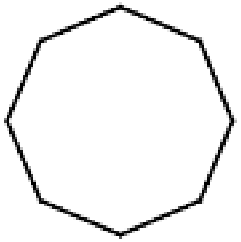


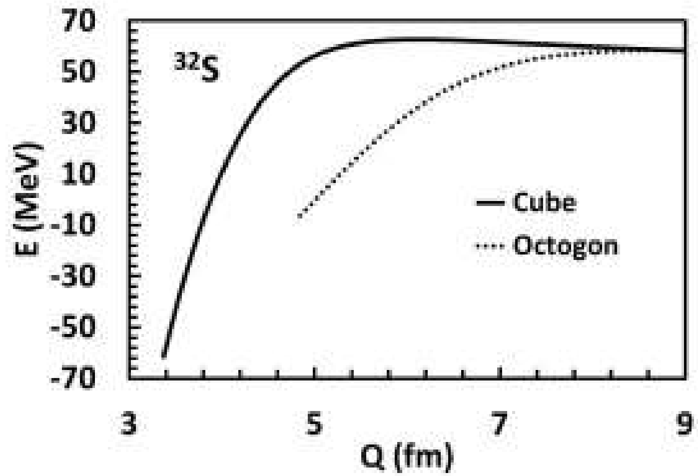


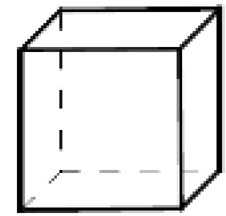
$Q_{6\alpha} = 28.48$ MeV

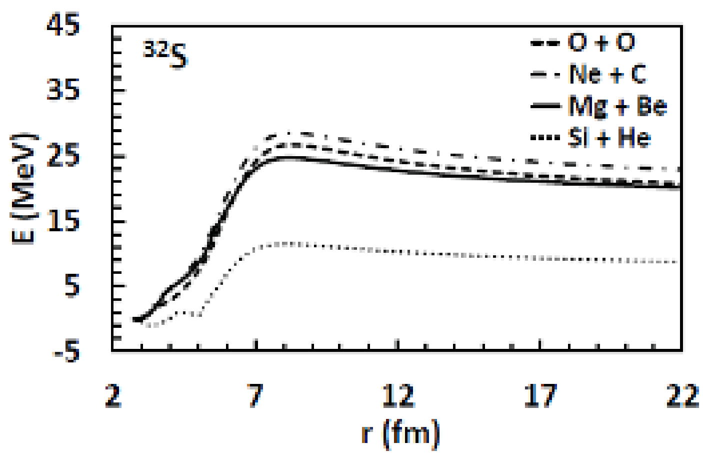
Reaction	^{20}Ne+^{4}He	^{12}C+^{12}C	^{16}O+^{8}Be	^{8}Be+^{8}Be+^{8}Be	^{10}B+^{4}He+^{10}B
Qreaction (MeV)	9.32	13.93	14.14	28.76	40.46

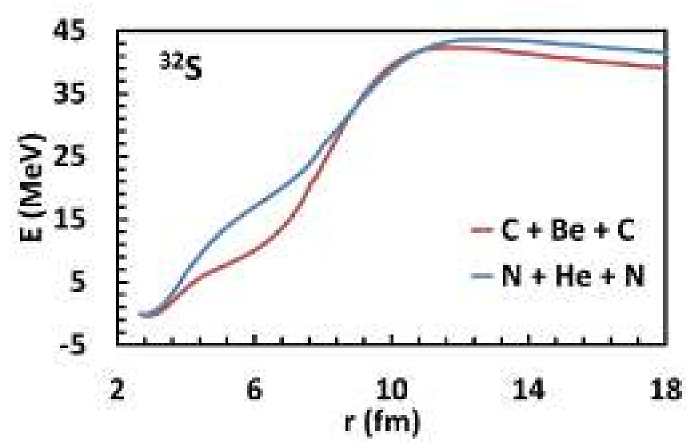
^{32}S nucleus











Reaction	$^{28}Si+^{4}He$	$^{16}O+^{16}O$	$^{24}Mg+^{8}Be$	$^{20}Ne+^{12}C$	$^{12}C+^{8}Be+^{12}C$	$^{14}N+^{4}He+^{14}N$
$Q_{reaction}$ (MeV)	6.95	16.54	17.02	18.97	30.96	34.17

$Q_{8\alpha} = 45.42$

rms radius

^{16}O	Square	Tetrahedron	Linear config.
rms radius (fm) Exp : 2.70 fm	2.83	2.54	4.15
Q_{elec} ($e.fm^2$)	-49.17 (Oblate)	0	
^{20}Ne	Pentagon	Trigonal bipyramid	Square pyramid
rms radius (fm) Exp : 3.01 fm	3.29	2.76	2.79
Q_{elec} ($e.fm^2$)	-89.63 (Oblate)	41.29 (Prolate)	-29.73 (Oblate)
^{24}Mg	Hexagon	Octahedron	
rms radius (fm) Exp : 3.06 fm	3.79	2.85	
Q_{elec} ($e.fm^2$)	-149.75 (Oblate)	0	
^{32}S	Octogon	Cube	
rms radius (fm) Exp : 3.26 fm	4.85	3.37	
Q_{elec} ($e.fm^2$)	-345.3 (Oblate)	0	

Conclusion

- A Liquid Drop Model previously used to describe smoothly the transition between two-(or three) body and one-body shapes in entrance and exit channels of nuclear reactions has been used to determine the potential barriers governing the evolution of the light nuclei : ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg and ^{32}S.
- The energy of these nuclei viewed as planar and three dimensional clusters of N-alphas in contact have been compared, as well as their rms radius and moment. These calculations suggest that an oblate equilateral triangular configuration is compatible with the ground state shape of ^{12}C and a prolate almost aligned shape for the excited Hoyle state shape. The three dimensional shapes are favored for the heavier nuclei.
- From ^{16}O the combination of an alpha and the daughter nucleus leads always to the lowest Q value. The proximity energy plays a main role to determine the energy of these quasimolecular nuclear configurations.

(International Symposium on Physics of Unstable Nuclei 2014, Ho Chi Minh City)

Thank you for your attention