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COLLINS ASYMMETRY IN FIELD IONIZATION OF HYDROGEN¹

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Abstract

An effect similar to the Collins asymmetry is found in the ionization of a hydrogen atom by a static electric field \mathbf{E} . When the initial electron possesses an orbital angular momentum $\langle \mathbf{L} \rangle$ transverse to the field, the mean transverse velocity $\langle \mathbf{v}_T \rangle$ of the final electron points in the direction of $\mathbf{E} \times \langle \mathbf{L} \rangle$. However $\langle \mathbf{L} \rangle$ is oscillating in time due to the linear Stark effect, making $\langle \mathbf{v}_T \rangle$ oscillate.

Introduction. An atom can be ionized by a sufficiently strong static electric field \mathbf{E} thanks to the tunnel effect. This process has a strong similarity with the production of a quark-antiquark pair ($q\bar{q}$) in a QCD string. If the initial electron has an orbital angular momentum perpendicular to \mathbf{E} , the average transverse velocity $\langle \mathbf{v}_T \rangle$ should be nonzero and in the direction of $\langle \mathbf{L} \rangle \times \mathbf{F}$, where $\mathbf{F} = -e\mathbf{E}$ is the external force [1]. We refer to it as the $\mathbf{v} \cdot (\mathbf{L} \times \mathbf{F})$ asymmetry. The mechanism (Fig.1-left) looks like the *string* + 3P_0 mechanism (Fig.1-right) of hyperon polarization [2] and Collins effect [1, 3].

At variance with the string + 3P_0 mechanism, the *Schwinger mechanism* of $q\bar{q}$ pair creation yields no $\mathbf{v} \cdot (\mathbf{L} \times \mathbf{F})$ asymmetry [1]. Thus the question of such an asymmetry in string breaking remains open. It is at least instructive to study it in atomic physics.

1 Behavior of an H atom in an external electric field

We consider an hydrogen atom in an static electric field $\mathbf{E} = -(F/e)\hat{\mathbf{z}}$. At small F the *linear* Stark effect just splits the n^{th} energy level in $2n - 1$ sublevels separated by² $\omega = 3nF/2$. *Stark states* are the eigenstates of $H_0 - Fz = \mathbf{p}^2/2 - 1/r - Fz$ in the $F \rightarrow 0$ limit. For large enough F ionization by tunneling becomes important and Stark states move into resonances of complex energy $E = E_R - i\gamma/2$. Using the parabolic coordinates $\xi=r-z$, $\eta=r+z$, $\varphi=\arg(x+iy)$, their wave functions have the separable form [4]

$$\Psi = \xi^{-1/2} \Phi(\xi) \eta^{-1/2} \chi(\eta) e^{im\varphi} \quad (1)$$

where $\Phi(\xi)$ verifies

$$\partial^2 \Phi / \partial \xi^2 + [E/2 + Z_\xi/\xi - (m^2 - 1)/(4\xi^2) - F\xi/4] \Phi(\xi) = 0. \quad (2)$$

and $\chi(\eta)$ an analogous equation with $F \rightarrow -F$ and $Z_\xi \rightarrow Z_\eta = 1 - Z_\xi$. Stark states are labeled $|n_\xi, n_\eta, m\rangle$, where n_ξ and n_η are the numbers of nodes of $\Phi(\xi)$ and $\chi(\eta)$, linked

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²In this paper we use atomic units: $\hbar/(m_e\alpha c) = 0.0529$ nm for length, $\hbar/(m_e\alpha^2 c^2) = 2.42 \cdot 10^{-17}$ s for time, $m_e\alpha^2 c^2 = 27,2$ eV for energy and $m_e^2\alpha^3 c^2/\hbar = 5.14 \cdot 10^9$ eV/cm for force.

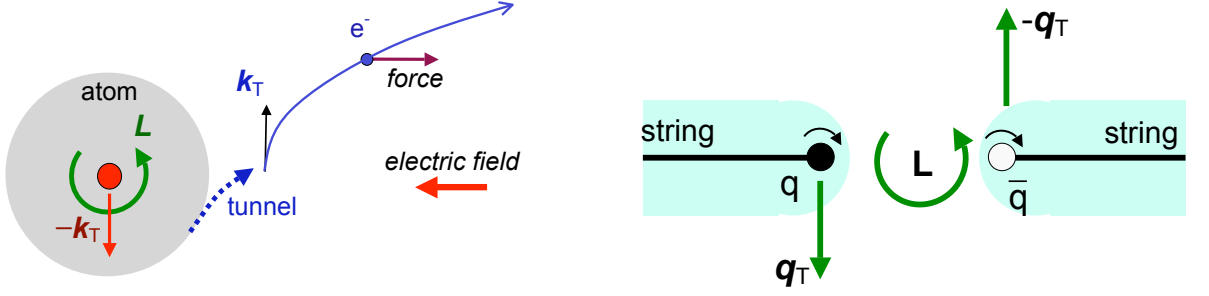


Figure 1: Left: semi-classical motion of the electron extracted from the hydrogen atom by a strong field \mathbf{E} , when the electron is initially in a $L_y = +1$ state. Right: String + 3P_0 mechanism correlating the transverse momentum and the transverse polarization of a quark created in string decay [1, 3].

by $n_\eta + n_\xi + |m| + 1 = n$ and fixing $Z_\xi = (n + n_\xi - n_\eta)/(2n)$. With the change of variables $\sqrt{\xi/n} e^{i\varphi} = \hat{x} + i\hat{y}$, $\hat{\Phi}(\hat{x}, \hat{y}) \equiv \xi^{-1/2} \Phi(\xi) e^{im\varphi}$ is the wave function of a 2-dimensional harmonic oscillator of angular momentum m and energy $\epsilon_\xi = 2nZ_\xi = 2n_\xi + |m| + 1$.

L_\perp oscillations. Stark states are also eigenstates of A_z , where \mathbf{A} is the Laplace-Runge-Lenz-Pauli vector

$$\mathbf{A} = \mathbf{r}/r + (\mathbf{L} \times \mathbf{p} - \mathbf{p} \times \mathbf{L})/2. \quad (3)$$

For $F=0$, $\langle A_z \rangle = 2\langle z \rangle / (3n^2) = (n_\eta - n_\xi)/n$. For $F \neq 0$ the transverse components (L_x, L_y) and (A_x, A_y) are not conserved. Starting from a L_y eigenstate, $\langle L_y \rangle$ oscillates in quadrature with $\langle A_x \rangle$, as pictured in Fig.2, with the period $2\pi/\omega$. Let us take as an example the initial state $|n=2, L_y=+1\rangle$, whose wave function is

$$\Psi(\mathbf{r}, t=0) = 8^{-1} \pi^{-1/2} (z + ix) e^{-r/2} = 0.5(|010\rangle - |100\rangle + i|001\rangle + i|00-1\rangle). \quad (4)$$

At $t \neq 0$ it evolves as

$$\Psi(t) = 0.5 e^{it/8} (e^{+i\omega t} |010\rangle - e^{-i\omega t} |100\rangle + i|001\rangle + i|00-1\rangle) \quad (5)$$

$$= e^{it/8} \left[\cos^2 \frac{\omega t}{2} |L_y=+1\rangle - \sin^2 \frac{\omega t}{2} |L_y=-1\rangle + \frac{i}{\sqrt{2}} \sin(\omega t) |l=0\rangle \right]. \quad (6)$$

Thus the atom oscillates between three L_y eigenstates.

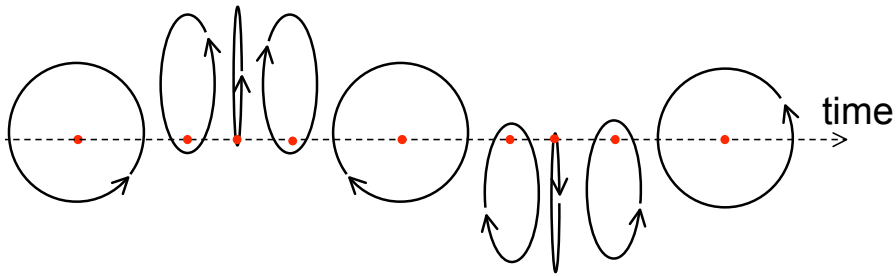


Figure 2: Classical picture of the Stark oscillations of L_y and A_x .

2 Tunneling amplitudes

The external force is confining in ξ and changes $\Phi(\xi)$ only little. Tunneling bears on $\chi(\eta)$. The wave function at large η describes the escaped electron. Using, as in Ref. [4], the JWKB method to lowest order in F , one obtains for the state $|i\rangle \equiv |n_\xi, n_\eta, m\rangle$

$$\Psi_i(\mathbf{r}, t)_{\eta \rightarrow \infty} \simeq a_i \hat{\Phi}_i(\hat{x}, \hat{y}) \exp\{(-i\delta E_i - \gamma_i/2)t'\} B(\eta, t). \quad (7)$$

a_i is the tunneling amplitude normalized to $|a_i|^2 = \gamma_i$, δE_i is the Stark shift, $\hat{\Phi}_i(\hat{x}, \hat{y})$ is the 2-D oscillator wave function normalized to $\langle \hat{\Phi}_i | \hat{\Phi}_i \rangle = 1$ and

$$B(\eta, t) = (4F\eta^3)^{-1/4} \exp\left[(i/3)\sqrt{F}(\eta - \eta_F)^{3/2} + it/8 + 5i\pi/4\right]. \quad (8)$$

$\eta_F \equiv 1/(n^2F)$ is near the tunnel exit and $t' \equiv t - \sqrt{(\eta - \eta_F)/F}$ is the classical electron exit time. For $n=2$ the amplitudes are

$$\begin{aligned} a_1 &\equiv a_{010} = iq a_{00+1}, \\ a_2 &\equiv a_{00+1} = a_{00-1} = 2^{-5/2} F^{-1} \exp[-1/(24F)], \\ a_3 &\equiv a_{100} = a_{00+1}/(iq), \end{aligned} \quad (9)$$

with $q = e^{-3/2}/\sqrt{2F}$. The widths $\gamma_i = |a_i|^2$ agree with Slavjanov's result [5].

3 $\mathbf{v} \cdot (\mathbf{L} \times \mathbf{F})$ asymmetry for the initial state $|n=2, L_y = +1\rangle$

With the initial state (4) the escaped electron density is, according to (5,7-8),

$$|\Psi(\mathbf{r}, t)|_{\eta \rightarrow \infty}^2 = (4F\eta^3)^{-1/2} \left| \hat{\Phi}(\hat{x}, \hat{y}, t') \right|^2 \quad (10)$$

with $\eta \simeq 2z$, $(\hat{x}, \hat{y}) \simeq (x, y)/\sqrt{2nz}$. In the $n=2$ case,

$$\hat{\Phi}(t') = 0.5 \left\{ a_1 \hat{\Phi}_{010} e^{(i\omega - \gamma_1/2)t'} - a_3 \hat{\Phi}_{100} e^{(-i\omega - \gamma_3/2)t'} + ia_2 (\hat{\Phi}_{00+1} + \hat{\Phi}_{00-1}) e^{-\gamma_2 t'/2} \right\}. \quad (11)$$

$|\Psi(\mathbf{r}, t)|^2$ looks like the density of a classical electron cloud falling freely in the force field \mathbf{F} . An electron leaving the tunnel at time t' with the transverse velocity \mathbf{v}_\perp follows the parabola of fixed $(\hat{x}, \hat{y}) \simeq \mathbf{v}_\perp/\sqrt{2F}$. The interference between even- and odd- m terms of $\hat{\Phi}(t')$ yields the $\mathbf{v} \cdot (\mathbf{L} \times \mathbf{F})$ asymmetry, which is t' -dependent. A measure of it is

$$A(t') \equiv \langle v_x \rangle / \Delta v_x = \langle \hat{\Phi}(t') | \hat{x} | \hat{\Phi}(t') \rangle / \sqrt{\langle \hat{\Phi}(t') | \hat{x}^2 | \hat{\Phi}(t') \rangle}; \quad (12)$$

$$A(t'=0) = 8^{1/2} (q^2 + 8 + 3q^{-2})^{-1/2}. \quad (13)$$

Like L_y , $A(t')$ changes sign at the Stark frequency, giving the "crawling snake" of Fig.3 [6].

Conclusion. This study shows that the $\mathbf{v} \cdot (\mathbf{L} \times \mathbf{F})$ effect does exist in field ionisation, but is oscillating in time. Several constraints make its search challenging:

- Radiative transition may compete with field ionization.
- The initial asymmetry $A(0)$ is small if the $|a_i|$'s differ too much (see Eqs.13 and 9).

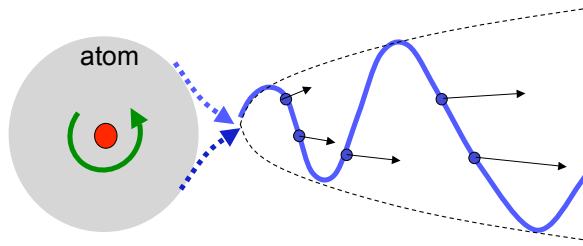


Figure 3: "Crawling snake" motion of $\langle x \rangle$ versus z of the escaping electron. As t grows the undulations move to the right.

- $A(t')$ is fast oscillating, therefore one may only measure its time-averaged $\langle A \rangle$. This one is large only if $\gamma_i \gtrsim \omega$, so that ionization is faster than oscillation.

These constraints are satisfied with a large enough field. In the $n=2$ case this field is too strong to be produced in laboratory. Hopefully, our results can be generalized to large n (Rydberg states), where the required field scales like n^{-4} [7]. The \mathbf{v}_T distribution can be measured by the *photoelectron imaging* techniques [8, 9].

Our formulae, obtained at lowest order in F , cannot be applied at the required field. Accurate numerical methods are given in [10, 11]. Nevertheless the above conclusions should remain qualitatively correct.

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