



**HAL**  
open science

## Aligned neutron–proton pairs in $N \sim Z$ nuclei

P. van Isacker

► **To cite this version:**

P. van Isacker. Aligned neutron–proton pairs in  $N \sim Z$  nuclei. NCPP 2011: Nordic Conference on Nuclear Physics 2011, 2011, Stockholm, Sweden. 014042 (3pp), 10.1088/0031-8949/2012/T150/014042 . in2p3-00714947

**HAL Id: in2p3-00714947**

**<https://in2p3.hal.science/in2p3-00714947v1>**

Submitted on 6 Jul 2012

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Aligned neutron–proton pairs in $N \sim Z$ nuclei

P Van Isacker

Grand Accélérateur National d'Ions Lourds, CEA/DSM–CNRS/IN2P3  
BP 55027, F-14076 Caen Cedex 5, France

**Abstract.** It is argued that  $N \sim Z$  nuclei with  $90 \leq A \leq 100$  can be interpreted in terms of aligned neutron–proton pairs with angular momentum  $J = 2j$  and isospin  $T = 0$ . Based on this observation, a version of the interacting boson model is formulated in terms of isoscalar high-spin bosons. To illustrate its possible use, the model is applied to the  $21^+$  isomer in  $^{94}\text{Ag}$ .

## 1. Introduction

One of the goals of radioactive-ion beam facilities is the uncovering of collective effects due to isoscalar ( $T = 0$ ) neutron–proton (n–p) pairing. In contrast to the usual isovector ( $T = 1$ ) pairing, where the orbital angular momenta and the spins of two nucleons are both antiparallel (*i.e.*,  $L = 0$  and  $S = 0$ ), isoscalar pairing requires the spins of the nucleons to be parallel ( $S = 1$ ), resulting in a total angular momentum  $J = 1$ . Collective correlation effects conceivably might occur as a result of isoscalar n–p pairing (and of pairs with  $J > 1$ ) but have resisted so far experimental confirmation [1].

Recently, the idea of a pair correlation effect was proposed by Blomqvist, as described in Ref. [2], when neutrons and protons are confined to a high- $j$  orbit. The idea is to interpret the structure of low-energy states of  $N \sim Z$  nuclei in terms of aligned n–p pairs coupled to maximum angular momentum  $2j$ . Currently,  $N \sim Z$  experiments are approaching  $^{100}\text{Sn}$ , and concern nuclei such as  $^{92}\text{Pd}$  [2] and  $^{96}\text{Cd}$  [3], to which Blomqvist's scheme can be applied since  $1g_{9/2}$  supposedly is the dominant orbit in this mass region.

In Ref. [4] Blomqvist's proposal has been examined with specific reference to the nuclei  $^{96}\text{Cd}$ ,  $^{94}\text{Ag}$ , and  $^{92}\text{Pd}$ , corresponding to four, six, and eight holes with respect to the  $^{100}\text{Sn}$  core, respectively. In addition to the aligned-pair assumption of Blomqvist the work of Ref. [4] was based on the supplementary hypothesis that the pairs behave as bosons and therefore effectively proposed a description of  $N \sim Z$  nuclei in terms of a (non-standard) interacting boson model (IBM) [5].

In this contribution I illustrate, with the particular example of the  $21^+$  isomer in  $^{94}\text{Ag}$ , how an IBM description in terms of isoscalar high-spin bosons may elucidate structural issues of  $N \sim Z$  nuclei in this mass region.

## 2. Aligned isoscalar pairs as bosons

Only a summary of results will be given in this section, referring for full details to Ref. [4]. The study consisted of two separate parts: (i) the analysis of shell-model wave functions of  $^{96}\text{Cd}$  in terms of aligned n–p pairs and (ii) the mapping of shell-model onto corresponding boson states for  $^{96}\text{Cd}$ ,  $^{94}\text{Ag}$ , and  $^{92}\text{Pd}$ .

(i) For a variety of shell-model interactions appropriate for this mass region, it was found that the  $^{96}\text{Cd}$  shell-model states can be well represented in terms of isoscalar n–p pairs with  $J = 2j$  (so-called  $B$  pairs). This conclusion came with two caveats. Firstly, the study [4] did not address (at least not in sufficient detail) the question whether  $1g_{9/2}$  is a dominant orbit in this mass region but rather *presupposed* that it is. Secondly, the  $8^+$  yrast state in  $^{96}\text{Cd}$  *cannot* be written in terms of two  $B$  pairs. This should have observational consequences in the form of loss of E2 collectivity between the yrast states of this nucleus.

(ii) An analysis of shell-model eigenstates for more than four nucleons is a challenging problem which has been studied by use of the multi-step shell model [6]. It is simpler, and at the same time instructive, to extend the analysis toward higher particle number through standard boson mapping techniques [7, 8]. It was found, again with some caveats (for full details see Ref. [4]), that the complicated spectroscopy of the nuclei  $^{96}\text{Cd}$ ,  $^{94}\text{Ag}$ , and  $^{92}\text{Pd}$  can, to a large extent, be accounted for with an interacting boson model containing a single type of boson with angular momentum  $\ell = 9$  (a so-called  $b$  boson, whence  $b$ -IBM).

## 3. The $21^+$ isomer in $^{94}\text{Ag}$

Not much is known experimentally about  $^{94}\text{Ag}$  except for the presence of two isomers, with tentative spin-parity assignments  $7^+$  (presumably the lowest  $T = 0$  state) and  $21^+$ , the latter at 6.7(5) MeV above the ground state [9]. The shell-model energy of both these states is reproduced with  $b$ -IBM to within less than 100 keV [4], and it can thus be expected that the latter model provides a good approximation to the former one. This can be demonstrated explicitly for the  $21^+$  isomer, as I now proceed to show.

In a shell-model description where three neutrons and three protons are placed in the  $1g_{9/2}$  orbit, the  $21^+$  state is stretched and therefore unique. In  $b$ -IBM this state arises from the coupling of three  $b$  bosons with  $\ell = 9$  to total angular momentum  $J = 21$ . The number of independent states that can be coupled to total angular momentum  $J$  arising from  $n$  bosons, each with individual angular momentum  $\ell$ , is given by the sum  $\sum_v d_v^{(\ell)}(J)$  (with  $v = n, n-2, \dots, 1$  or  $0$ ) where the multiplicity  $d_v^{(\ell)}(J)$  is known in terms of an integral over characters of the orthogonal algebras  $\text{SO}(2\ell + 1)$  and  $\text{SO}(3)$  [10, 11],

$$d_v^{(\ell)}(J) = \frac{i}{2\pi} \oint_{|z|=1} \frac{(z^{2J+1} - 1)(z^{2v+2\ell-1} - 1) \prod_{k=1}^{2\ell-2} (z^{v+k} - 1)}{z^{\ell v + J + 2} \prod_{k=1}^{2\ell-2} (z^{k+1} - 1)} dz. \quad (1)$$

By virtue of Cauchy's theorem  $d_v^{(\ell)}(J)$  is obtained as the negative of the residue of the integrand in (1). One finds  $d_3^{(9)}(21) = 2$  and  $d_1^{(9)}(21) = 0$  and, therefore, two independent

boson states with  $J = 21$  can be constructed, one of which must be spurious. As explained in Ref. [4], the spurious state is eliminated in  $b$ -IBM by taking an infinitely repulsive interaction between two  $b$  bosons coupled to angular momentum  $J = 18$ . Furthermore, since the coefficients of fractional parentage (CFPs) needed in a three-particle problem are known [12], the following energy expression for the  $J = 21$  state can be derived:

$$E(b^3; 21^+) = 3\epsilon_b + \frac{6851}{20155}\nu_{12}^b + \frac{15488}{21545}\nu_{14}^b + \frac{1212882}{624805}\nu_{16}^b, \quad (2)$$

in terms of the two-boson interaction matrix elements  $\nu_\lambda^b \equiv \langle b^2; \lambda | \hat{V}_b | b^2; \lambda \rangle$ , and where  $\epsilon_b$  is the energy of the  $b$  boson. By virtue of the mapping method, the boson energy  $\epsilon_b$  and the two-boson interaction matrix elements can be expressed in terms of the shell-model two-body interaction matrix elements  $\nu_\lambda^f \equiv \langle (1g_{9/2})^2; \lambda | \hat{V}_f | (1g_{9/2})^2; \lambda \rangle$ . From the mapping of the two-particle system one finds  $\epsilon_b = \nu_9^f$ . From the mapping of the four-particle system, which also can be carried out analytically, one derives

$$\begin{aligned} \nu_{12}^b &= \frac{1218}{69355}\nu_3^f + \frac{63423}{138710}\nu_4^f + \frac{29957}{63050}\nu_5^f + \frac{109881}{53350}\nu_6^f + \frac{1148337}{2358070}\nu_7^f \\ &\quad + \frac{15231}{31525}\nu_8^f + \frac{10893}{535925}\nu_9^f, \\ \nu_{14}^b &= \frac{868}{8515}\nu_5^f + \frac{1953}{1310}\nu_6^f + \frac{46251}{57902}\nu_7^f + \frac{1977}{1310}\nu_8^f + \frac{2211}{22270}\nu_9^f, \\ \nu_{16}^b &= \frac{8}{17}\nu_7^f + 3\nu_8^f + \frac{9}{17}\nu_9^f. \end{aligned} \quad (3)$$

By inserting these results into Eq. (2), one finds

$$\begin{aligned} E_b(21^+) &= \frac{22134}{3707825}\nu_3^f + \frac{1152549}{7415650}\nu_4^f + \frac{1347751953}{5740387250}\nu_5^f + \frac{8606149749}{4857250750}\nu_6^f + \\ &\quad + \frac{354940047213}{214690483150}\nu_7^f + \frac{1561553973}{220784125}\nu_8^f + \frac{15411107094}{3753330125}\nu_9^f. \end{aligned} \quad (4)$$

This is an approximate expression since it is derived by use of a boson mapping (whence the index ‘b’). To what extent it is wrong therefore yields an idea about the reliability of the boson approximation.

The exact fermionic energy expression for three neutrons and three protons in a  $j = 9/2$  orbit, can be derived with standard techniques involving CFPs [12]. Since the  $J = 21$  state is unique, its energy  $E_f(j^6 JT)$  is the matrix element  $\langle j^6 JT | \hat{V}_f | j^6 JT \rangle = \sum_\lambda a_\lambda \nu_\lambda^f$ , with the coefficients  $a_\lambda$  given by

$$a_\lambda = 15 \sum_{\alpha' J' T'} [j^4(\alpha' J' T') j^2(\lambda) JT | \{ j^6 JT \}^2]. \quad (5)$$

For  $j = 9/2$ ,  $J = 21$  and  $T = 0$ , the following expression results:

$$E_f(21^+) = \frac{21}{65}\nu_5^f + \frac{21}{10}\nu_6^f + \frac{645}{442}\nu_7^f + \frac{69}{10}\nu_8^f + \frac{717}{170}\nu_9^f. \quad (6)$$

Since the highest allowed angular momentum for two neutrons and two protons in a  $j = 9/2$  orbit is  $J = 16$ , only interaction matrix elements  $\nu_\lambda^f$  with  $\lambda \geq 5$  can contribute to the energy of the  $J = 21$  state in the  $3n-3p$  system. This rule is obviously obeyed in Eq. (6) but violated in Eq. (4). It is seen, however, that the coefficients of  $\nu_3^f$  and

$\nu_4^f$  are rather small in the latter expression, indicating that the boson approximation is reasonably accurate.

The coefficients  $a_\lambda$  in the energy expression  $E_f(j^n JT) = \sum_\lambda a_\lambda \nu_\lambda^f$  for a unique  $n$ -particle shell-model state with angular momentum  $J$  and isospin  $T$ , satisfy the identities

$$\begin{aligned} \sum_{\lambda=0}^{2j} a_\lambda &= \frac{n(n-1)}{2}, \\ \sum_{\lambda=0}^{2j} \lambda(\lambda+1)a_\lambda &= J(J+1) + j(j+1) \times n(n-2), \\ \sum_{\substack{\lambda=0 \\ \text{even}}}^{2j} 2a_\lambda &= T(T+1) + \frac{3}{4}n(n-2). \end{aligned} \quad (7)$$

These identities are valid for the coefficients in Eq. (6). It is of interest to note that they are also *exactly* satisfied by the coefficients in Eq. (4). This reflects the conservation of particle number, angular momentum and isospin in the shell model, and the preservation of these quantum numbers under the mapping procedure.

#### 4. Concluding remark

More results, for example concerning the moments of the  $21^+$  isomer, can be derived to test the validity of the boson approximation. Of more interest will be a similar analysis of the  $7^+$  isomer in  $^{94}\text{Ag}$ : its structure in the shell model, even when confined to the  $1g_{9/2}$  orbit, is complicated with 30 components in the  $JT$  scheme and more than 500 in the  $m$  scheme. In contrast, the number of independent components is only three in terms of  $B$  pairs (or  $b$  bosons) which allows for a better understanding of the structure of the  $7^+$  isomer. A preliminary analysis shows, for example, that its main component involves two  $b$  bosons coupled to intermediate angular momentum 16 which is then coupled with the last boson to total  $J = 7$ . This problem is currently under further study.

#### References

- [1] Warner D D, Bentley M, and Van Isacker P 2006 *Nature Phys.* **2** 311.
- [2] Cederwall B et al. 2011 *Nature* **469** 68; also Qi C, Blomqvist J, Bäck T, Cederwall B, Johnson A, Liotta R J, and Wyss R this issue.
- [3] Cederwall B, Wadsworth R, and de France G 2011 *GANIL proposal E623*.
- [4] Zerguine S and Van Isacker P 2011 *Phys. Rev. C* **83** 064314.
- [5] Iachello F and Arima A 1987 *The Interacting Boson Model* (Cambridge University Press, Cambridge).
- [6] Xu Z X, Qi C, Blomqvist J, Liotta R J, and Wyss R 2012 *Nucl. Phys. A* **877** 51.
- [7] Otsuka T, Arima A, and Iachello F 1978 *Nucl. Phys. A* **309** 1.
- [8] Skouras L D, Van Isacker P, and Nagarajan M A 1990 *Nucl. Phys. A* **516** 255.
- [9] Mukha I et al. 2005 *Phys. Rev. Lett.* **95** 022501; Cerny J et al. 2009 *Phys. Rev. Lett.* **103** 152502.
- [10] Weyl H 1939 *The Classical Groups* (Princeton University Press, Princeton).
- [11] Gheorghe A and Raduta A A 2004 *J. Phys. A: Math. Gen.* **37** 10951.
- [12] Talmi I 1993 *Simple Models of Complex Nuclei* (Harwood Academic Publishers, Chur, Switzerland).