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The role of superfluidity in nuclear incompressibilities

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Nuclei are propitious tools to investigate the role of the superfluidity in the compressibility of a Fermionic system. The centroid of the Giant Monopole Resonance (GMR) in Tin isotopes is predicted using a constrained Hartree-Fock Bogoliubov approach, ensuring a full self-consistent treatment. Superfluidity is found to favour the compressibility of nuclei. Pairing correlations explain why doubly magic nuclei such as ²⁰⁸Pb are stiffer compared to open-shell nuclei. Fully self-consistent predictions of the GMR on an isotopic chain should be the way to microscopically extract both the incompressibility and the density dependence of a given energy functional. The macroscopic extraction of K_{sym} , the asymmetry incompressibility, is questioned. Investigations of the GMR in unstable nuclei are called for. Pairing gap dependence of the nuclear matter incompressibility should also be investigated.

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The effect of the superfluidity on the compressibility of a Fermionic system remains an open question. Superfluidity initially referred to a system with a dramatic drop of its viscosity [1]: it could be suspected that a super-fluid would be easier to compress than a normal fluid. This question has been investigated in Fermionic atoms traps, by studying the frequency of the compression mode with respect to the scattering length. Experimentally some increase of the frequency may be observed in the weak pairing regime [2], but this signal remains to be confirmed. Theoretically both microscopic and hydrodynamical investigations show no variation of the compression mode between the normal and the superfluid phases [3], but the analysis is complicated by the temperature change between the two phases. In nuclear physics, the study of the role of superfluidity in the compressibility can also be performed: the isoscalar Giant Monopole Resonance (GMR) is a compression mode, allowing to probe for related superfluid effects. Ideal tools are especially isotopic chains, where pairing effects are evolving from normal (doubly-magic) nuclei to superfluid (open-shell) ones [4]. Moreover, the incompressibility of nuclear matter is a basic parameter in calculations describing neutron stars or supernovae, where superfluid effects are known to occur [5].

Constraining the nuclear incompressibility modulus K_∞ with experimental data on the Giant Monopole Resonance is a longstanding problem. The first relevant approaches have been performed following the work of Blaizot and Pearson [6, 7]: only microscopic predictions of the GMR compared with the data could validate the K_∞ value of the functional which was used. Using a fully self-consistent approach such as the Random Phase Approximation (RPA) [8] or the constrained Hartree-Fock (CHF) [9] method, the GMR data on ²⁰⁸Pb currently provides $K_\infty \simeq 230$ MeV in non-relativistic approaches [10] whereas $K_\infty \simeq 260$ MeV is obtained for relativistic one [11]. The roots of this puzzle between the two approaches is still an open question, but it has been shown that the neutron-proton asymmetry dependence of the incompressibility, denoted as K_{sym} , plays also a role, as well as the density dependence of the functional [12]: a value of $K_\infty \simeq 250$ MeV could be extracted from ²⁰⁸Pb data, using a non-relativistic CHF method with a modified density dependence of the functional. Therefore, it should be noted that K_∞ cannot be extracted from the measurement of the GMR in a single nucleus: several parts of the functional are tested simultaneously, namely K_∞ , K_{sym} and its density dependence.

A previous study on the role of superfluidity on nuclear incompressibility has been performed, finding a negligible effect [13], but the theoretical approach was not self-consistent. Indeed self-consistency is crucial since pairing effects are expected to be small in the GMR: this high energy mode is mainly built from particle-hole configurations located far from the Fermi level, where pairing does not play a major role. However giant resonances are known to be very collective [14] and pairing can still have a sizable effect on the GMR properties: around 10 % on the centroid, which is the level of accuracy of present analysis on the extraction of K_∞ [10, 11]. This requires the advent of accurate microscopic models in the pairing channel, such as fully self-consistent Quasiparticle Random Phase Approximation (QRPA) [15, 16], achieved only recently. Experimentally, the measurement of the GMR on an isotopic chain facilitates the study of superfluidity on the GMR properties [17], and the possibility to measure the GMR in unstable nuclei emphasizes this feature [18].

It is therefore necessary to go towards the measurement of the GMR on several nuclei, such as an isotopic chain. The overused method of precise GMR measurements in a single nucleus, such as ²⁰⁸Pb, may not be the relevant approach. Other nuclei have been used such as ⁹⁰Zr and ¹⁴⁴Sm. Indeed when considering the available GMR data from which the K_∞ value has been extracted, ²⁰⁸Pb is stiffer than the Sn, Zr and Sm nuclei: K_∞ is about 20 MeV larger, both in non-relativistic and in relativistic approaches [10, 11]. The question may not be "why Tin are so soft

?" [19] but rather "why ^{208}Pb is so stiff?"

Recently the GMR was measured on the stable Tin isotopic chain (from ^{112}Sn to ^{124}Sn) [17]. Once again it has been noticed that it is not possible to describe the GMR both in Sn and in Pb with the same functional, Tin being softer than Pb [15, 19]. In the non relativistic case, fully self-consistent QRPA calculations on Sn isotopes lead to $K_\infty \simeq 215$ MeV. The relativistic DDME2 parameterisation using QRPA describes well the GMR in the Sn isotopes [20], but predict a low value of the GMR in ^{208}Pb compared to the experiment, as can be seen on Fig. 8 of Ref. [11]. On the contrary, a recent relativistic functional describes well the ^{208}Pb GMR, but systematically overestimates the Sn GMR values of about 1 MeV [21]. Finally, attempts have been performed in order to describe Sn GMR data with relativistic functionals having a lower incompressibility and hence different density dependence and K_{sym} value [22]. Once again the ^{208}Pb and Tin GMR cannot be described at the same time: this puzzling situation is due to the higher value of K_∞ extracted from ^{208}Pb , compared to Tin, Sn and Zr nuclei.

In Ref. [15] it has been found that including pairing effects in the description of the GMR allows to explain part of the Sn softness : pairing may decrease the predicted centroid of the GMR of few hundreds of keV, located at ~ 16 MeV. This is sufficient to change by about 10 MeV the extracted value of the incompressibility of nuclei K_A , defined as [6]:

$$E_{GMR} = \sqrt{\frac{\hbar^2 K_A}{m \langle r^2 \rangle}} \quad (1)$$

where m is the nucleon mass and $\langle r^2 \rangle$ is the ground-state mean square radius.

In this work we follow up this idea and show that the consequences of superfluidity on nuclear incompressibility may solve the above mentioned puzzle. It should be noted that pairing is vanishing in the doubly magic ^{208}Pb nucleus, unlikely the other nuclei. It is necessary to use a fully microscopic method including an accurate pairing approach. In order to predict the GMR in a microscopic way we use the constrained HF method, extended to the full Bogoliubov pairing treatment (CHF(B)). The CHF(B) method has the advantage to very precisely predict the centroid of the GMR using the m_{-1} sumrule [9, 12]. The whole residual interaction (including spin-orbit and Coulomb terms) is taken into account and this method is by construction the best to predict the GMR centroid [12]. Introducing the monopole operator as a constraint, the m_{-1} value is obtained from the derivative of the mean value of this operator. The m_1 sumrule is extracted from the usual double commutator, using the Thouless theorem [23]. Finally the GMR centroid is given by $E_{GMR} = \sqrt{m_1/m_{-1}}$. All details on the CHF method can be found in [9, 10].

The extension of the CH method to the CHF(B) case uses the HFB approach in coordinate space [24]. Skyrme functionals and a zero-range surface pairing interaction are used in this work. The magnitude of the pairing interaction is adjusted so to describe the trend of the neutron pairing gap in Tin isotopes. This interaction is known to describe a large variety of pairing effects in nuclei [25].

Fig. 1 displays the GMR energy obtained from the Sn measurements (times $A^{1/3}$ to correct for the slow lowering of the GMR with the nuclear mass [14]). Microscopic CHF(B) predictions using two functionals are also shown: SLy4 [26] ($K_\infty=230$ MeV, which describes well the Pb GMR data), and SkM* [27] ($K_\infty=215$ MeV). Without pairing, the SLy4 interaction overestimates the Sn GMR data. Pairing effects (CHF(B) calculations) decrease the centroid of the GMR, getting closer to the data. This confirms the results of [15], where a self-consistent HFB+QRPA approach was used to describe the Tin data. It should be noted that using a less general BCS approach for the pairing channel increases the GMR energy [13, 15, 28]. This is due to the problematic treatment of high energy single-particle states in the BCS model. It is therefore important to use the full HFB approach to correctly describe pairing effects on the GMR, especially for the pairing residual interaction. Fig. 1 shows, as expected, that SkM* predictions with pairing effects are in better agreement with the Sn data.

The striking feature of Fig. 1 is the peak of the GMR centroid, located at the doubly magic ^{132}Sn nucleus, using the CHF(B) predictions. This indicates that pairing effects should be considered to describe the behaviour of nuclear incompressibility, and that vanishing of pairing make the nuclei stiffer to compress, confirming our previous statement on the stiffness of ^{208}Pb . The importance of pairing effect can be understood in a simple way: since the nuclear incompressibility is defined as the second derivative of the energy functional at saturation density [6], there is no obvious reason why the pairing terms of the functional would play no role in the nuclear incompressibility. Nuclear incompressibility is indeed very close from a residual interaction (as a second derivative of the energy functionals with respect to the density), and it is known that pairing effects are relevant in residual interactions [25]. This is straightforward in nuclear matter where K_∞ is expressed from the F_0 Landau parameter [29]. However, on Fig. 1, the GMR centroid is shifted to lower energies for more neutron-rich nuclei than ^{132}Sn , also because of the appearance of a soft $L=0$ mode, predicted in QRPA calculations beyond ^{132}Sn [30].

To further investigate the role of pairing on nuclear incompressibility, Fig. 2 displays K_A (defined by Eq. (1)) with respect to the average pairing gap calculated using the HFB approach, from ^{112}Sn to ^{132}Sn . A clear correlation

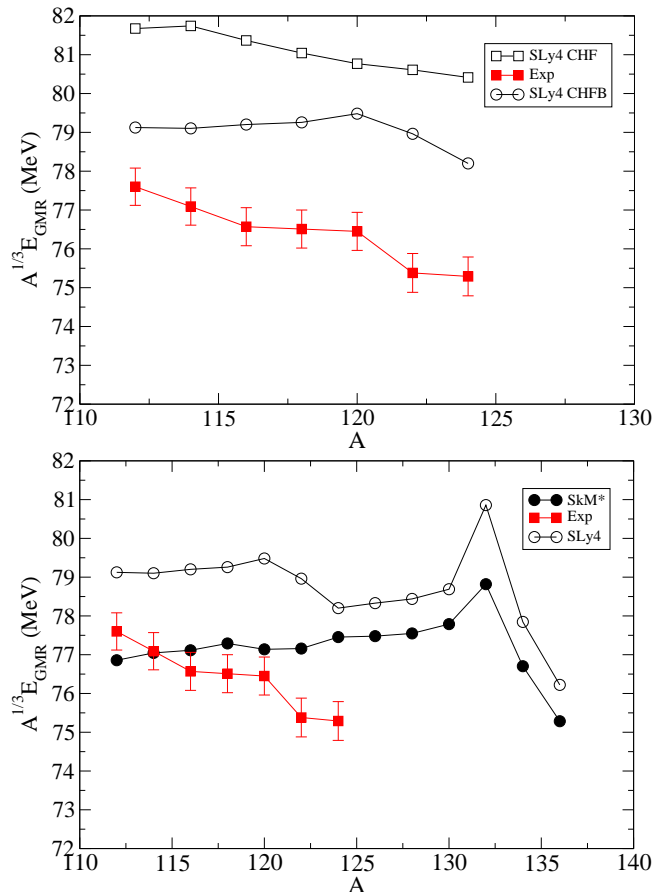


FIG. 1: *Top:* Excitations energies of the GMR in stable $^{112-124}\text{Sn}$ isotopes calculated with constrained HF and constrained HFB methods and the SLy4 interaction, compared to the data. *Bottom:* Same for $^{112-136}\text{Sn}$ isotopes, using the CHFB method and the SLy4 and SkM* interactions

is observed: the more superfluid the nuclei, the lower the incompressibility. Hence it may be easier to compress superfluid nuclei. This may be the first evidence of the role of superfluidity on the compressibility of a Fermionic system. A possible interpretation is that Cooper pairs can modify bulk properties, as known from nuclear physics phenomenology [4].

The decrease of incompressibility in superfluid nuclei raises the question of a similar effect in infinite nuclear matter: for now, incompressibility is given independently from the pairing part of the functional. However, considering present results, equations of state used for neutron star and supernovae predictions should take into account pairing to provide their incompressibility value. The comparison with GMR data shows, as mentioned above, that the functional as a whole (including pairing effects) is probed. The question of the behaviour of K_{∞} with respect to the pairing gap is raised: it seems clear from nuclear data that nuclei incompressibility K_A decreases with increasing pairing gap. This should be investigated in nuclear matter.

It should be noted that recent attempts have been performed to extract the K_{sym} value, and its corresponding quantity in nuclei (K_{τ}): the incompressibility in the Sn isotopic chain has been studied using the macroscopic (liquid drop) formula of nuclei incompressibility K_A derived by Blaizot [6, 17, 31]. However the effect of pairing demonstrated above shows that the current macroscopic approach may not be well designed: on Fig. 2, pairing effects induce ~ 10 MeV change on the nuclear incompressibility K_A . Hence the macroscopic expression of nuclei incompressibility K_A should be extended to these terms, since the appropriate definition of nuclei incompressibility is the second derivative of the microscopic energy functional [6]. Presently, the microscopic approach is more relevant to extract K_{sym} : the extraction of K_{∞} , K_{sym} and the density dependence of the functional are related, as stated in Ref. [12]. The GMR data on isotopic chains should be used on this purpose.

It is not possible to describe the GMR centroid of both ^{208}Pb and other nuclei with the same functional, as stated above. The puzzle of the stiffness of ^{208}Pb may come from its doubly magic behaviour. In Fig. 1 there is a sharp peak at doubly magic ^{132}Sn , and it would be very interesting to measure the GMR in this unstable nucleus. It should be

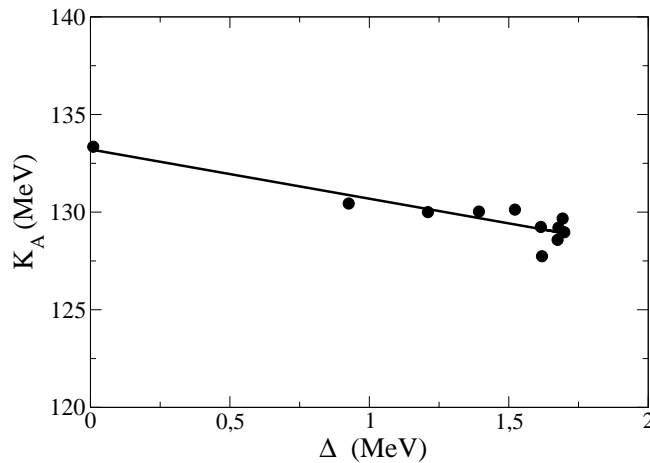


FIG. 2: Nuclear incompressibilities K_A in $^{112-132}\text{Sn}$ isotopes calculated with the CHF method and the SkM* interaction, as a function of the pairing gap Δ predicted by the HFB calculation.

noted that such experiments are now feasible [18]. A possible explanation of the ^{208}Pb stiffness is that the experimental data of E_{GMR} is especially increased in the case of doubly magic nuclei, as observed in ^{208}Pb compared to the GMR data available in other nuclei (such as the Tin isotopic chain). This difficulty to describe with a single functional both doubly magic and other nuclei has already been observed on the masses, namely the so-called "mutually enhancement magicity" (MEM), described in [32, 33]: functionals designed to describe masses of open-shell nuclei cannot predict the masses of doubly magic nuclei such as ^{132}Sn and ^{208}Pb , which are systematically more bound than predicted. In order to consider MEM, it may be necessary to take into account quadrupole correlation effects due to the flatness of the potentials for open-shell nuclei [34]. K_A being related to the second derivative of the energy with respect to the density, it would be useful to find a way to predict the GMR beyond QRPA by taking into account such quadrupole correlations. This may solve the current puzzle of the stiffness of ^{208}Pb . It should be noted that it would also be relevant to measure the GMR on the Pb isotopic chain in order to provide a similar analysis than the one on the Sn nuclei.

In conclusion, it is shown that superfluidity favours the compressibility of nuclei, using a fully microscopic CHF approach on the Tin isotopic chain. This may be the first evidence of a sizable effect of superfluidity on the compressibility of a Fermionic system. Pairing effects should be described using a full microscopic HFB treatment. Doubly magic nuclei exhibit a specific increase of the GMR energy, due to the collapse of pairing. ^{208}Pb is therefore the "anomalous" data compared to the others. It is not possible to disentangle pairing interaction from the equation of state when providing the nuclear matter value of K_∞ . Indeed the pairing gap dependence on the nuclear matter incompressibility should be investigated, since it is shown that incompressibility decreases with increasing pairing gap in nuclei. Additional theoretical investigations are called for in order to predict the GMR including the mutually enhancement magicity effect. The macroscopic extraction of K_{sym} may be ill-defined and should be extended to include pairing effects. Experimentally, measurements of the GMR in unstable nuclei should be performed in doubly magic ^{132}Sn , as well as extending the measurement on the Sn and Pb isotopic chains.

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